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Risk Return Optimization with Different Risk Aggregation Strategies

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Survey

Risk Return Optimization with Different Risk Aggregation Strategies

Gaia Serraino[*)], Ursula Theiler[+)], Stanislav Uryasev[#)]

1. Motivation and Actual Requirements on Banks' Risk Management
2. Optimization Problem Formulation
3. Case Study on Bank Portfolio Optimization
4. Summary

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1. Motivation and Actual Requirements on Banks‘ Risk Management

Regulatory Requirements on Banks’ Risk Management

Basel Committee of Banking Supervision - Capital Adequacy Framework (2004)

“Basel II”

Pillar 1	Pillar 2	Pillar 3
Minimum Capital Requirements	**Qualitative Requirements**	**Market-discipline**
Capital Standards for Market, Credit and Operational Risk	Internal Capital Adequacy Assessment Process (ICAAP)	

Quantitative Standards: “Regulatory risk ≤ Regulatory Capital”	**Internal Approaches and Models** “Internal risk ≤ Economic Capital”

Internal Capital Adequacy Assessment Process:
“Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels”.
(Basel Committee of Banking Supervision, June 2004, § 725)

1. Motivation and Actual Requirements on Banks‘ Risk Management

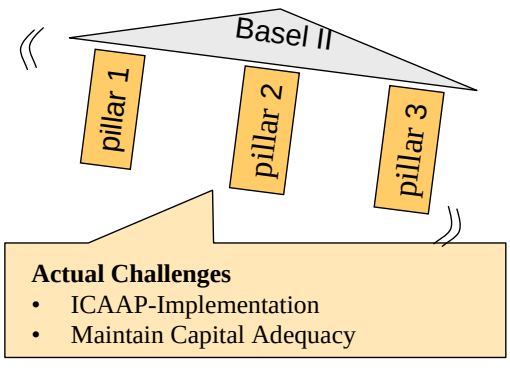
Actual Challenges of Bank Management

- Financial crisis, market turbulences, increasing risks,
- High competition, low return margins,
- *Quantitative and qualitative* regulatory requirements on bank management.

Competitive Necessity:
- **Implement Risk-/Return-efficient management of overall bank portfolio**
- **Apply Risk-/Return optimization techniques on bank portfolio!**

Maximize
Total expected risk adjusted returns
(or risk return ratios)

Constraints:
- Constraint risks by regulatory capital (meet quantitative standards of **pillar 1**)
- Constraint risks by economic capital (meet internal risk constraints of **pillar 2**)
- …

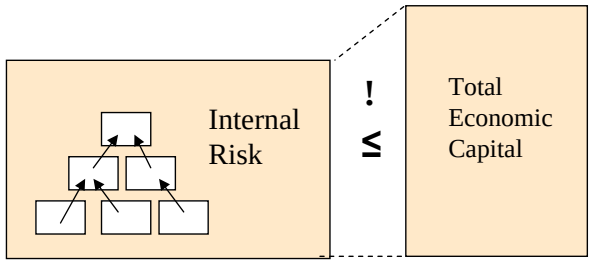


1. Motivation and Actual Requirements on Banks‘ Risk Management

Practical questions:
Implementation of enterprise wide risk management (pillar 2)

- How to measure risk appropriately?
- How to aggregate risk on portfolio level appropriately?
- How to identify risk-/return-efficient overall portfolios?
- How to allocate capital efficiently? …

”Internal Capital Adequacy”



Our contribution:

- We suggest a **risk-/return optimization approach for a bank portfolio**, which applies
  - different methodologies of risk measurement and risk aggregation
  - in the context of actual regulatory requirements.
- We analyze the effects of different approaches of internal risk aggregation.
- We suggest a systematic approach for risk strategy formulation and capital allocation

2. Optimization Model Formulation

General Problem Statement:

Objective function: subject to constraints	maximize expected returns
Constraint Set 1:	Internal risk ≤ Economic capital,
Constraint Set 2:	Regulatory risk ≤ Regulatory capital, Constraints on the regulatory capital components (tier 1, 2, 3),
Constraint Set 3:	Exposure constraints

Variation of risk measures and risk aggregation strategies

Related Research / Previous Work:

- Measures for risk quantification and optimization [1,2,3,5,15,16,17,20,21].
- Generalization of classical portfolio selection approaches [13,17,18],
- Application of portfolio optimization approaches to Credit and Bank Portfolios [22, 23].

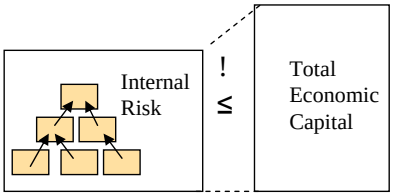
2. Optimization Model Formulation

Optimization Model Specifications ("Constraint Set 1")

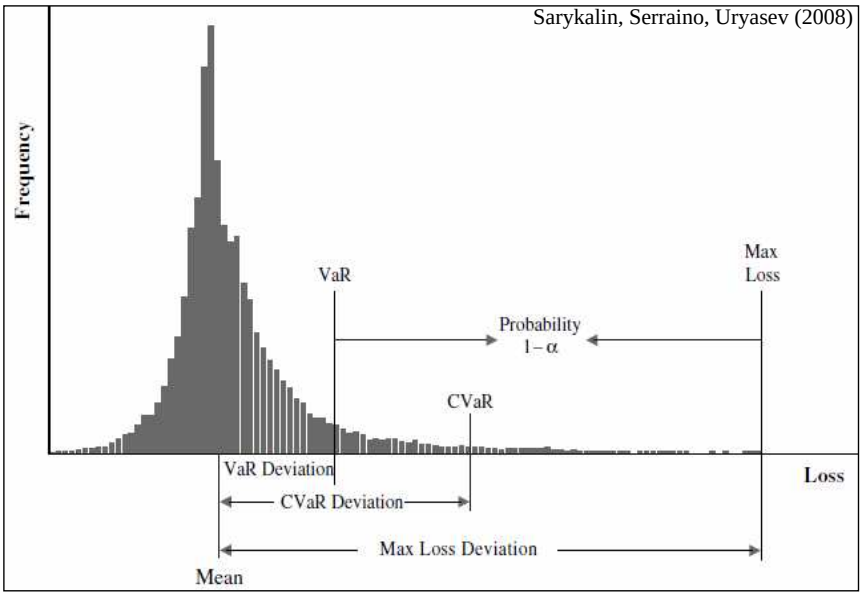
Internal Constraint

*How to **measure** risk appropriately ?*

Constraint Set 1:
"Internal Capital Adequacy"



Sarykalin, Serraino, Uryasev (2008)



Pro VaR	**Con VaR**
Acceptance and implementation in practice, …	Methodological weaknesses (lack of sub-additivity,…)
Con CVaR	**Pro CVaR**
Lower acceptance in practice (increasing), …	Axiomatic foundation, desirable properties, (coherence, convexity,…)

2. Optimization Model Formulation

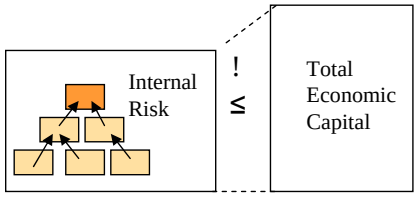
Optimization Model Specifications ("Constraint Set 1")

Internal Constraint

*How to **aggregate** risk appropriately ?*

Typically, a financial institution calculates the loss distributions for different risk types and / or units on a standalone basis and then aggregates the economic capital partial amounts on portfolio level. [9]

"Internal Capital Adequacy"



Aggregation Approaches: [9,19]

Let E_i denote the economic capital for unit *i*, *i=1,…n*, E_{total} *the overall economic capital.*

Worst Case Approach

Measure risk from marginal loss distributions,
aggregate standalone capital amounts by the sum of the economic capital amounts.

$$E_{total} = \sum_{i=1}^{n} E_i \qquad (1)$$

Hybrid Approach

Measure risk from marginal loss distributions,
aggregate standalone capital amounts by correlation model.
Let $R=(\rho_{ij})$, i,j=1,…,n denote the correlation matrix:

$$E_{total} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} E_i E_j \rho_{ij}} \qquad (2)$$

Integrated Approach

Measure risk from joint loss distribution to calculate total economic capital E_{total}.

- aggregate marginal loss distributions by copula approaches,
- or (if available) use joint empirical distributions

2. Optimization Model Formulation

Optimization Model Specifications ("Constraint Set 1")
Internal Constraint

Optimization problem variations

We consider three cases for the constraint set 1
"**Internal risk** ≤ Economic capital"

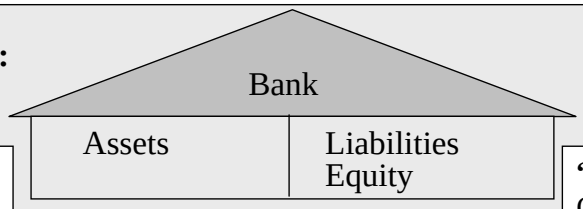
Problem 1	**Problem 2**	**Problem 3**
Worst Case Approach	***Hybrid Approach***	***Integrated Approach***
Worst case aggregation	Correlation aggregation, Internal risk is limited on aggregated level by correlation matrix approach	Joint *empirical* distribution, economic capital constraint on portfolio level, (based on historical data bootstrapping)

For each problem we apply **VaR deviation** (case a)) and **CVaR deviation** (case b)) to measure economic capital amounts E_i.

2. Optimization Model Formulation

Optimization Model Specifications ("Constraint Set 2")
Regulatory Constraint

Basic Idea (Pillar 1):



"Regulatory Risk": Regulatory standards to measure different risks

≤ !

"Regulatory Capital": Capital Reserves to cover losses, defined by regulations

Regulatory capital reserves must cover regulatory risks!

Solvability Coefficient:

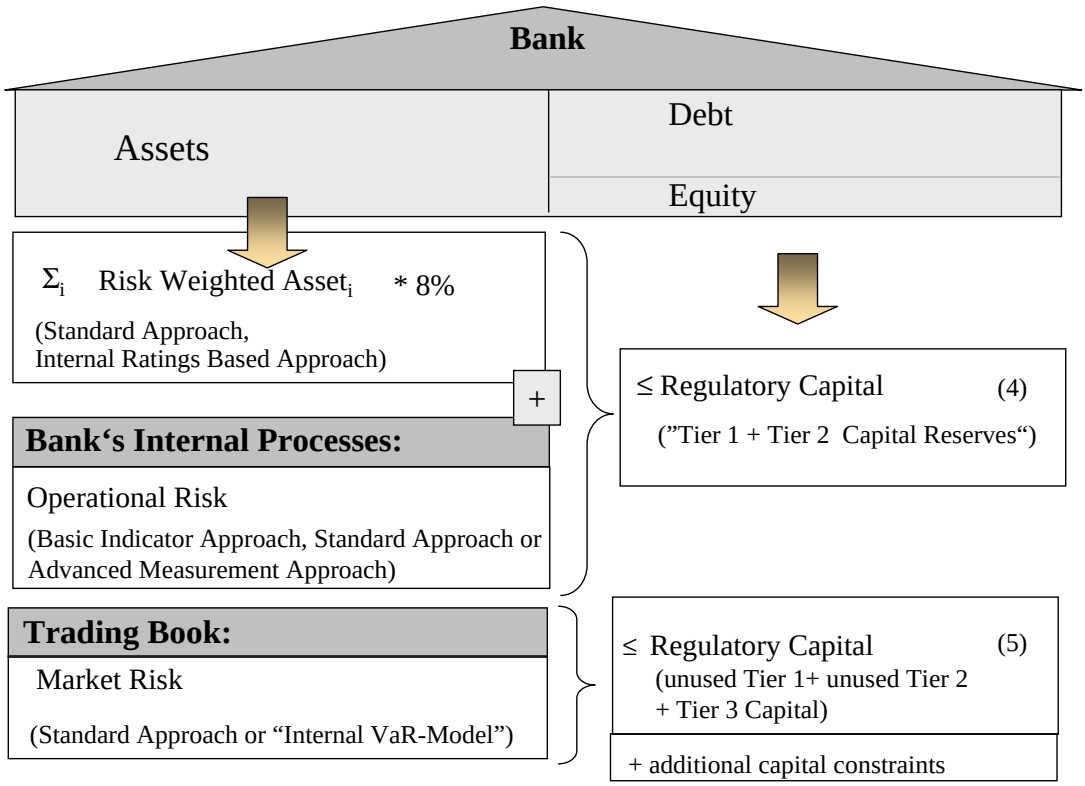
$$8\% \leq \frac{\text{Regulatory Capital}}{\text{Regulatory Risk}} \Leftrightarrow \text{Regulatory Risk}*8\% \leq \text{Regulatory Capital} \quad (3)$$

Basel I:
= Credit Risk (1988)
+ Market Risk (1996)

Basel II (2004):
= Credit Risk ("new" risk weights)
+ Market Risk
+ Operational Risk

2. Optimization Model Formulation

Optimization Model Specifications ("Constraint Set 2")
Regulatory Constraint



2. Optimization Model Formulation

Model Formulation

Objective: Maximize estimated portfolio return $\max \sum_{i=1}^{n} r_i x_i$ (6)

subject to

Constraint Set 1 – Internal Constraint on Total Economic Capital

Problem 1 (Worst Case Approach)

$$a) \sum_{j=1}^{m} VaR\_Dev_{\alpha\_\text{int}}(\tilde{\mathbf{x}}_j) \le C_{ec\_total\_risk} - C_{ec\_other\_risk} \quad (7)$$

$$b) \sum_{j=1}^{m} CVaR\_Dev_{\alpha\_\text{int}}(\tilde{\mathbf{x}}_j) \le C_{ec\_total\_risk} - C_{ec\_other\_risk}$$

Problem 2 (Hybrid Approach)

$$a)\, VaR\_Dev_{\alpha\_int}(\tilde{\mathbf{x}}_j) \le x^j_{ec\_cap}\,, \quad b)\, CVaR\_Dev_{\alpha\_int}(\tilde{\mathbf{x}}_j) \le x^j_{ec\_cap}\,. \quad (8)$$

$$\sqrt{(x^1_{ec\_cap},\ldots,x^m_{ec\_cap}) \begin{pmatrix} 1 & \ldots & \rho_{1m} \\ \ldots & \ldots & \ldots \\ \rho_{m1} & \ldots & 1 \end{pmatrix} \begin{pmatrix} x^1_{ec\_cap} \\ \ldots \\ x^m_{ec\_cap} \end{pmatrix}} \le C_{ec\_total\_risk} - C_{ec\_other\_risk}\,. \quad (9)$$

Problem 3 (Integrated Approach)

$$a)\, VaR\_Dev_{\alpha}(\mathbf{x}) \le C_{ec\_total\_risk} - C_{ec\_other\_risk}\,, \quad b)\, CVaR\_Dev_{\alpha}(\mathbf{x}) \le C_{ec\_total\_risk} - C_{ec\_other\_risk}\,. \quad (10)$$

2. Optimization Model Formulation

Model Formulation (ctd.)

Constraint Set 2 – Regulatory Capital (pillar 1)

Balance equation for the regulatory capital covering bank book (credit risk):

$$\sum_{i=1}^{n\_bb} w_i^{reg\_cr} x_i^{bb} + reg\_op\_risk = x_1^a + x_2^a, \tag{11}$$

Balance equation for the regulatory capital covering trading book (market risk):

$$\sum_{i=1}^{n\_tb} w_i^{reg\_spec} x_i^{tb} + w^{reg\_mult\_mr} VaR_{99\%}(x^{tb}), \tag{12}$$
$$\leq C_{tier\_3} + (C_{tier\_1} - x_1^a) + (C_{tier\_2} - x_2^a),$$

Constraint limiting unused Tier-2 and used Tier-3 capital vs. unused Tier-1 capital

$$x_3^a + (C_{tier\_2} - x_2^a) \leq 2.5(C_{tier\_1} - x_1^a), \tag{13}$$

Bounds on used Tier 1, Tier 2, and Tier 3 capital:

$$0 \leq x_k^a \leq C_{tier\_k},\ k = 1,2,3. \tag{14}$$

2. Optimization Model Formulation

Model Formulation (ctd.)

(15)

Constraint Set 3 – Exposure Constraints

Upper and lower bounds on decision variables (exposures)

$$l_i \leq x_i^{tb} \leq u_i,\ \ i = 1,...,n\_tb$$
$$l_i \leq x_i^{bb} \leq u_i,\ \ i = 1,...,n\_bb$$

Constant investment amount over all assets

$$\sum_{i=1}^{n\_bb} x_i^{bb} + \sum_{i=1}^{n\_tb} x_i^{tb} = Total\_Inv \quad . \tag{16}$$

3. Case Study on Bank Portfolio Optimization

Proceeding:

3.1 Portfolio setup and Hypothesis Formulation

3.2 Optimizations - **Step 1:**

- Run optimization problem (P) for all three problem variations Problem 1,2, and 3 in the Constraint Set 1,
- Use **VaR Deviation measure (case a))**
- Analyze exposures and risk return ratios of optimal portfolios

3.3 Optimizations – **Step 2:**

- Run optimization problem (P) for all three problem variations Problem 1,2, and 3 in the Constraint Set 1,
- Use **CVaR Deviation measure (case b))**
- Analyze exposures and risk return ratios of optimal portfolios

3.4 Comparison of optimal portfolios and conclusion

3. Case Study on Bank Portfolio Optimization

3.1 Portfolio Setup and Hypothesis Formulation

We consider typical Universal Bank Portfolio:

	Index	**Expo-sure**	**Rating**	**Basel II risk weight**	**Lower Exposure Bounds**	**Upper Exposure Bounds**
		(bill. US $)			(in % of initial exposure)	
Credit Portfolio						
Securities Government debt	Markit iBoxx $ Domestic Sovereigns & Sub-Sov.AAA	220	AAA	0%	-20%	+20%
Commercial and Industrial Credit	Markit iBoxx $ Domestic Corporates BBB 1-3Y	130	BBB	100%	-20%	+20%
Real Estate Loans	EPRA/NAREIT US Index	330	BB	35%	-20%	+20%
Inter bank Loans	Markit iBoxx $ Eurodollar Financials AA 1-3Y	40	AA	20%	-20%	+20%
Market Portfolio						
Equity Position USA	DJ Industrial Average	100		2%	-100%	+100%
Equity Position Europe	DJ EURO STOXX 50	100		2%	-100%	+100%

3. Case Study on Bank Portfolio Optimization

3.1 Portfolio Setup and Hypothesis Formulation

Assumptions on Constraint Set 1: Economic Capital

-We are using time series for market and credit assets from January 3, 2000 to December 26, 2007. We are using 1 year log returns for both market and credit assets.

-We are applying 99% confidence level for VaR Deviation for the internal risk constraint in the Step 1 of the case study.

-We are applying 97,5% confidence level for CVaR Deviation for the internal risk constraint in the Step 2, as for the initial portfolio we have

$$99\%\text{-VaR} \approx 97.5\% \text{ CVaR} \approx 129.44. \tag{17}$$

Assumptions on Constraint Set 2: Regulatory Capital

-For credit risk we use the risk weights according to Standardized Approach.

-For market risk we apply the internal VaR model.

-We do not consider operational risk.

-We assume that the bank only uses tier 1 and tier 2 capital.

Assumptions on Constraint Set 1 and 2:

-The upper bounds on regulatory and economic capital are set equal to the corresponding capital use of the initial portfolio.

-We assume all other risks under pillar 1 and pillar 2, which are treated as constants in the optimization model formulation (P), equal to zero.

3. Case Study on Bank Portfolio Optimization

3.1 Portfolio Setup and Hypothesis Formulation

Hypothesis:

From Problems 1 to 3

- risk adjusted returns and risk adjusted performance ratios
- *will increase at the given level of economic capital,*

as portfolio effects are taken into consideration in a more risk adjusted way.

Risk Adjusted Performance Ratio:
"Return on Risk Adjusted Capital"

$$\text{RORAC} := \frac{\text{Expected Return}}{\text{Risk (Economic Capital)}} \tag{18}$$

3. Case Study on Bank Portfolio Optimization

3.2 Optimizations - Step 1

Exposures of Initial Portfolio and Optimal Solutions in Step 1
(with 99.0% - VaR Deviation)

	Initial Portfolio	**Problem 1**	**Problem 2**	**Problem 3**
Exposures (billion US $)		*Worst Case Approach*	*Hybrid Approach*	*Integrated Approach*
Government Debt Securities	220.00	264.00	264.00	229.91
Commercial and Industrial Credit	130.00	146.52	131.86	104.00
Real Estate Loans	330.00	299.67	311.83	396.00
Interbank Loans	40.00	48.00	48.00	32.00
Equity Position USA	100.00	161.81	164.31	158.09
Equity Position Europe	100.00	0.00	0.00	0.00
Total Exposure	**920.00**	**920.00**	**920.00**	**920.00**

3. Case Study on Bank Portfolio Optimization

3.2 Optimizations - Step 1

Risk Return Ratios of Optimal Portfolios

	Initial Portfolio	**Problem 1**	**Problem 2**	**Problem 3**
		Worst Case Approach	*Hybrid Approach*	*Integrated Approach*
Optimal Solutions in Step 1 (with 99.0% - VaR Deviation)				
Economic Capital (billion US $)	129.44	129.44	129.44	129.44
Estimated Return (billion US $)	21.70	20.66	20.90	22.14
RORAC := Est. Return / Econ. Capital	16.76%	15.96%	16.15%	17.11%

⇨ We observe increasing expected returns and increasing RORAC‘s from problem 1 over problem 2 to problem 3.

3. Case Study on Bank Portfolio Optimization

3.3 Optimizations - Step 2

Exposures of Initial Portfolio and Optimal Solutions in Step 2

(with 97.5% - CVaR Deviation)

	Initial Portfolio	Problem 1	Problem 2	Problem 3
Exposures (billion US $)		*Worst Case Approach*	*Hybrid Approach*	*Integrated Approach*
Government Debt Securities	220.00	264.00	264.00	227.73
Commercial and Industrial Credit	130.00	133.43	119.74	104.00
Real Estate Loans	330.00	396.00	321.88	391.17
Interbank Loans	40.00	48.00	48.00	32.00
Equity Position USA	100.00	78.57	166.37	165.10
Equity Position Europe	100.00	0.00	0.00	0.00
Total Exposure	**920.00**	**920.00**	**920.00**	**920.00**

3. Case Study on Bank Portfolio Optimization

3.4 Comparison of Optimal Portfolios and Conclusion

Risk Return Ratios of Optimal Portfolios

	Initial Portfolio	Problem 1	Problem 2	Problem 3
		Worst Case Approach	*Hybrid Approach*	*Integrated Approach*
Optimal Solutions in Step 1 (with 99.0% - VaR Deviation)				
Economic Capital (billion US $)	129.44	129.44	129.44	129.44
Estimated Return (billion US $)	21.70	20.66	20.90	22.14
RORAC := Est. Return / Econ. Capital	16.76%	15.96%	16.15%	17.11%

Optimal Solutions in Step 2 (with 97.5% - CVaR Deviation)				
Economic Capital (billion US $)	129.44	129.44	129.44	129.44
Estimated Return (billion US $)	21.70	20.90	21.11	22.17
RORAC := Est. Return / Econ. Capital	16.76%	16.15%	16.31%	17.13%

⇨ We observe increasing expected returns and increasing RORAC‘s from problem 1 over problem 2 to problem 3.

⇨ Expected returns and RORAC’s of optimal portfolios in step 2 are slightly higher.

3. Case Study on Bank Portfolio Optimization

3.4 Comparison of Optimal Portfolios and Conclusion

We compare the risk of the optimal portfolios of the tree problem variations in Step 1 and Step 2 at a lower confidence level:

95%-CVaR Deviation	Initial Portfolio	Problem 1	Problem 2	Problem 3
		Worst Case Approach	*Hybrid Approach*	*Integrated Approach*
Step 1 (VaR-Dev Optimization)	49.29	24.70	27.52	42.44
Step 2 (CVaR-Dev Optimization)	49.29	23.40	29.86	43.33

- In both, Step 1 and 2, the 95% CVaR Deviation is higher for Problem 3 then for problem 1 and 2,
- in each solution it is lower than 95% CVaR of the initial portfolio.
- This is an expected result because the risk aggregation model in Problem 3 is least conservative.

3. Case Study on Bank Portfolio Optimization

3.4 Comparison of Optimal Portfolios and Conclusion

Conclusion

- We found evidence for the initial hypothesis:
  - In both, Step 1 and Step 2,
    (under measures VaR Deviation and CVaR Deviation)
    - expected returns have been increased
      from problem 1 over problem 2 to problem 3,
    - RORAC ratios have been increased
      from problem 1 over problem 2 to problem 3.
  - We conclude that from problem 1 over 2 to 3 more efficient use is made of the economic capital.

- The case study illustrates
  that banks should use appropriate methodologies of risk integration,
  which allow to use capital resources most efficiently,
  in order to maximize profits and enforce their competitive position.

4. Summary

Main contributions

- We suggested an optimization approach for bank portfolio which considers internal and regulatory capital constraints in risk management.
- We applied novel methods of portfolio optimization, and analyzed different risk aggregation strategies and risk measures in the internal risk constraint.
- The practical application of the optimization approach can support
  - Risk-/return-efficient portfolio management,
  - Efficient capital use of different capital reserves in risk management,
  - Integration of internal and regulatory requirements.

Further research

- Need for further research and practical implementations for integrated risk measurement, modeling risk relations and capital allocation.
- Extend the case study for applications of advanced risk aggregation strategies.
- ...

Thank you for your attention! ☺

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Notations in the Optimization Model

Notation	Explanation
n_tb, n_bb	Number of assets in trading book and bank books, respectively
$n=n\_tb+n\_bb$	Total number of assets
$r=(r_1,\dots r_n)'$	Vector of estimated returns of assets
$x^{bb}=\begin{pmatrix} x_1^{bb} \\ \dots \\ x_{n\_bb}^{bb} \end{pmatrix}$	(Sub-)Vector of decision variables of the bank book
$x^{tb}=\begin{pmatrix} x_1^{tb} \\ \dots \\ x_{n\_tb}^{tb} \end{pmatrix}$	(Sub-)Vector of decision variables of the trading book
$x=\begin{pmatrix} x_1 \\ \dots \\ x_{n\_bb} \\ x_{n\_bb+1} \\ \dots \\ x_n \end{pmatrix}-\begin{pmatrix} x_1^{tb} \\ \dots \\ x_{n\_tb}^{tb} \\ x_1^{tb} \\ \dots \\ x_{n\_tb}^{tb} \end{pmatrix}-\begin{pmatrix} x^{bb} \\ \\ x^{tb} \end{pmatrix}$	Vector of decision variables, i.e., asset exposures $\tilde{x}_j$ the j-th subportfolio, j=1,…m
$\begin{pmatrix} 1 & \dots & \rho_{1m} \\ \dots & \dots & \dots \\ \rho_{m1} & \dots & 1 \end{pmatrix}$	Risk correlation matrix for sub-portfolios, j=1, …m.
$C_{ec\ other\ risk}$	Available economic capital for other risk (pillar 2)
$C_{ec\ total\ risk}$	Available economic capital for total risk (pillar 2)
$C_{reg\ tier\ i}, i=1,\dots,3.$	Available Components of Regulatory Capital, tier i, i=1,…,3
$x^j_{ec\_cap}, j=1,\dots, m$	Used economic capital in internal risk constraint for sub-portfolio j
$x_{reg\ tier\ i}, i=1,\dots,3.$	Used regulatory tier i, i=1,…,3 capital
reg_op_risk	Constant for regulatory capital for operational risk (Basic Indicator Approach)
$w_j^{reg\_cr}, j=1,\dots n\_bb$	Regulatory capital weights for credit assets of the bank book
$w_j^{reg\_sp}, j=1,\dots n\_bb$	Regulatory risk weights for market risk constraint: specific risk of assets
$w^{reg\_mult\_mr}$	Regulatory multiplication factor for VaR model
α_int	Confidence level for internal economic risk constraints (pillar 2)
$Total\_Inv$	Upper bound for overall investment exposures

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