



# Risk-return optimization with different risk-aggregation strategies

Risk-return optimization

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Stan Uryasev

*University of Florida, Gainesville, Florida, USA*

Ursula A. Theiler

*Risk Training, Kleinmachnow, Germany, and*

Gaia Serraino

*American Optimal Decisions, Inc., Austin, Texas, USA*

## Abstract

**Purpose** – New methods of integrated risk modeling play an important role in determining the efficiency of bank portfolio management. The purpose of this paper is to suggest a systematic approach for risk strategies formulation based on risk-return optimized portfolios, which applies different methodologies of risk measurement in the context of actual regulatory requirements.

**Design/methodology/approach** – Optimization problems to illustrate different levels of integrated bank portfolio management has been set up. It constrains economic capital allocation using different risk aggregation methodologies. Novel methods of financial engineering to relate actual bank capital regulations to recently developed methods of risk measurement value-at-risk (VaR) and conditional value-at-risk (CVaR) deviation are applied. Optimization problems with the portfolio safeguard package by American Optimal Decision (web site: [www.AOrDA.com](http://www.AOrDA.com)) are run.

**Findings** – This paper finds evidence that risk aggregation in Internal Capital Adequacy Assessment Process (ICAAP) should be based on risk-adjusted aggregation approaches, resulting in an efficient use of economic capital. By using different values of confidence level  $\alpha$  in VaR and CVaR, deviation, it is possible to obtain optimal portfolios with similar properties. Before deciding to insert constraints on VaR or CVaR, one should analyze properties of the dataset on which computation are based, with particular focus on the model for the tails of the distribution, as none of them is “better” than the other.

**Research limitations/implications** – This study should further be extended by an inclusion of simulation-based scenarios and copula approaches for integrated risk measurements.

**Originality/value** – The suggested optimization models support a systematic generation of risk-return efficient target portfolios under the ICAAP. However, issues of practical implementation in risk aggregation and capital allocation still remain unsolved and require heuristic implementations.

**Keywords** Financial risk, Risk assessment

**Paper type** Research paper

## 1. Introduction

New regulations are imposing high standards on internal risk management in financial institutions. In its accord, *International Convergence of Capital Measurement and Capital Standards – A Revised Framework*, the Basel Committee on Banking Supervision has set forth a new set of regulations on risk management for financial institutions (“Basel II”). The new regulations are based on three pillars: Pillar 1 consists of new minimum capital requirements, Pillar 2 enforces qualitative standards on risk



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management, and Pillar 3 requires risk management information disclosure, thus enforcing market discipline (Basel Committee on Banking Supervision, 2004). While in recent years, financial institutions have been focusing on the implementation of the quantitative requirements of Pillar 1, attention recently shifted towards implementation of Pillars 2 and 3 requirements. Pillar 2 of the new Basel Accord postulates four principles of qualitative requirements on banks' internal risk management ("supervisory review process"). The intention is to insure that banks have adequate capital to support all the risks in their ongoing business.

The first principle is also denoted as the Internal Capital Adequacy Assessment Process (ICAAP): "Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels"[1]. To meet these requirements, the banks must aggregate material risks and allocate economic capital against them to cover potential financial losses. The internal capital adequacy needs to be balanced on an integrated portfolio level. A risk strategy must ensure capital adequacy in the overall business. While the quantitative standards of Pillar 1 are clearly defined, many issues of internal risk management under Pillar 2 methodologically are still unsolved or are difficult to implement, frequently due to the lack of sufficient data support. Banks are challenged to apply appropriate methodologies for integrated risk measurement under Pillar 2.

We suggest an optimization model approach, which generates risk-return efficient portfolios with respect to internal (Pillar 2) and regulatory (Pillar 1) capital requirements, and illustrate its application by an example. For an optimized portfolio, we derive capital allocation and an efficient risk strategy, which is required by ICAAP (for a summary refer to Rosenberg and Schuermann, 2006). One important issue in this context is to aggregate different risk types: market, credit, and operational risk, which show considerable variations in their distributions. Different approaches of measuring integrated risk and aggregating different risk types have been developed and have become a major object of discussion. We consider different approaches of risk aggregation in the context of portfolio optimization and ICAAP implementation. The capital requirements under Pillar 1 thereby represent a minimum capital standard and a strict constraint that must be maintained continuously. Risk strategies frequently are derived by carrying forward actual portfolio compositions or heuristic allocations of economic capital or exposures by industry or branches. We suggest a systematic approach to derive consistent and efficient risk strategies from an optimal asset allocation.

The rest of the paper is organized as follows. Section 2 derives a general formulation of optimization models considering different approaches of risk aggregation and integrated risk measurement in ICAAP, Section 3 describes the case study, which applies the formulated optimization models, and Section 4 concludes the paper.

## **2. Formulation of optimization model**

### *2.1 Survey on optimization problem statement*

*2.1.1 Related research.* Capital regulations can be considered in the context of portfolio optimization. In this context, the banks' objective function takes the form of an expected utility function, or alternatively, the banks face a Markowitz (1952) mean-variance portfolio-selection problem with additional constraints. For a discussion refer to Furlong and Keeley (1990), Kim and Santomero (1988), Koehn

and Santomero (1980) and Rochet (1992). Empirical research has been conducted to analyze the effects of regulations on the optimal capital structure in the inter-action of regulation and bank management. Several studies were pursued in the option-pricing framework. From the existing literature no clear statement can be deduced as to how capital regulations impact banks' risk strategies.

In our approach, we apply novel methods of financial engineering to relate actual bank capital regulations to recently developed methods of risk measurement and portfolio optimization, as introduced by Rockafellar and Uryasev (2000). We extend the model for optimization of bank portfolios suggested by Theiler (2004) and apply the introduced risk-return management approach to an illustrative bank portfolio example. To optimize portfolios, we use the portfolio safeguard (PSG) package by American Optimal Decisions (www.AOrDa.com).

*2.1.2 General problem statement.* We formulate a one period optimization model that maximizes expected returns of the bank portfolio with Pillars 1 and 2 requirements. Decision variables are the asset exposures. The capital constraints take into consideration economic and regulatory capital limits. Here is the general structure of the optimization problem for the bank portfolio[2]:

- *Objective function.* Maximize expected returns subject to constraints.
- *Constraint Set 1.* Internal risk  $\leq$  economic capital.
- *Constraint Set 2.* Regulatory risk  $\leq$  regulatory capital constraints on the regulatory capital components (Tiers 1-3).
- *Constraint Set 3.* Exposure constraints.

## 2.2 Modeling regulatory requirements

*2.2.1 Modeling requirements on ICAAP (Pillar 2).* To meet ICAAP requirements, banks must measure relevant risks and allocate sufficient economic capital to cover them. The modeling of the internal capital adequacy raises a series of methodological questions about risk assessment, which financial institutions are implementing at different levels of sophistication. In the following, we focus on actual questions of integrated risk measurement and risk aggregation.

Modeling interactions of different risks: risk integration approaches. The issue of risk aggregation has recently become an area of study[3]. A financial institution typically calculates the loss distributions for different risk types or for a number of business units on a standalone basis. Then, it aggregates the loss distributions and calculates the total economic capital for the whole enterprise (Hull, 2007, p. 373). In industry practice, easier-to-implement approximations are widely used; however, more sophisticated approaches, such as copula-based methods, are increasingly discussed in financial theory.

Traditional approaches calculate different risks separately, without considering their interactions, and add risks to achieve an integrated risk measure. With this "worst-case approach," total economic capital for  $n$  different risks is the sum of the economic capitals for each risk considered on a standalone basis (Hull, 2007, p. 374):

$$\text{Tot\_Ec\_Cap} = \sum_{i=1}^n E_i. \quad (1)$$

It can be observed that a simple adding up of risks (marginal distributions for individual contributors) significantly overestimates risks (and consequently economic capital), which is not surprising as it assumes the perfect inter-risk correlation (Rosenberg and Schuermann, 2006).

The “normal distribution” approach is based on the assumption that loss distributions are normal (Hull, 2007, p. 374). The standard deviation of the total loss from  $n$  sources then is:

$$\sigma_{\text{total}} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \sigma_i \sigma_j \rho_{ij}}, \quad (2)$$

where:

$\sigma_i$  = the standard deviation of the loss from the  $i$ th source of risk.

$\rho_{ij}$  = the correlation between risks  $i$  and  $j$ .

In the parametric value-at-risk (VaR) approach, the economic capital can be calculated by a multiple of the standard deviation of the normal distribution, i.e. the 99 percent VaR can be obtained by multiplying the portfolio standard deviation by 2.326. However, the joint normality assumption of risk factors imposes a distribution tail that is much thinner than the empirically observed one. “Normal” tails may significantly underestimate economic capital. This approach is not acceptable when one or more marginal densities exhibit significant negative skewness or excess kurtosis (Rosenberg and Schuermann, 2006). We do not consider further this approach in the context of ICAAP because the capital adequacy may not be maintained in this case.

In the “hybrid approach,” risks are calculated on a standalone basis with possibly “heavy” tail distributions. Then, risks are aggregated by a correlation model, which combines marginal risks using the formula that would apply to an elliptical distribution (Rosenberg and Schuermann, 2006, p. 10; Hull, 2007, p. 375). Let  $\mathbf{R} = (\rho_{ij})$ ,  $i, j = 1, \dots, n$  denote the correlation matrix, such that  $\rho_{ij}$  denotes the correlation of risk  $i$  and  $j$ . The overall risk is then calculated as the square root of the product of the vector of economic capitals,  $\mathbf{E} = (E_1, \dots, E_n)'$  with the correlation matrix  $\mathbf{R}$ :

$$\text{Tot\_Ec\_Cap} = \sqrt{\mathbf{E}'\mathbf{R}\mathbf{E}} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n E_i E_j \rho_{ij}}. \quad (3)$$

This approach is exactly correct for the special case when the marginal return distributions are normal. Rosenberg and Schuermann (2006) demonstrate that this approach may correctly aggregate heaviness in the tails of the individual loss distributions. It is surprisingly accurate and seems to combine copula-based models quite well (Rosenberg and Schuermann, 2006, p. 3; Hull, 2007, p. 375).

Integrated risk models represent the most advanced method of risk integration. Different approaches are suggested to derive the overall portfolio distribution. Copula models aggregate the marginal loss risk distributions on the portfolio level. The essential idea of the copula approach is that a joint distribution can be factored into marginals and a dependence function called copula[4]. Simulation techniques are used

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at the portfolio level to derive the integrated portfolio loss risk distribution (Hull, 2007, p. 152).

Our formulations of Constraints Set 1 (internal risk) in the optimization problems take into consideration different approaches of risk aggregation: worst case, hybrid, and integrated approach.

Risk measures for integrated risk measurement. The ICAAP implementation needs appropriate risk measures on the integrated portfolio level. The discussion on risk measures has become a major object of research. The assumption of normally distributed returns, frequently assumed for market risk measurement, typically does not hold at the portfolio level, when different types of risk are aggregated. VaR risk measure is commonly applied in finance for market risk measurement. However, this measure has poor mathematical properties; in particular, it lacks sub-additivity if loss distributions are not normal. This means that portfolio diversification may increase risk, i.e. VaR of a combined portfolio may be higher than the sum of the VaRs of the sub-portfolios. VaR is a widely accepted risk measure and represents the industry standard of risk measurement. However, there is an increasing awareness of the problems that VaR may cause in risk measurement at the integrated portfolio level.

For a random variable  $X$  with continuous distribution function, conditional value-at-risk ( $\text{CVaR}_\alpha(X)$ ) equals the conditional expectation of  $X$  subject to  $X \geq \text{VaR}_\alpha(X)$ . The general definition of CVaR for random variables with a possibly discontinuous distribution function is more complex and can be found in Rockafellar and Uryasev (2002).

CVaR is sub-additive and is appropriate for risk measurement of any loss distribution[5]. In our case study, we consider both VaR and CVaR in the context of portfolio optimization. We use VaR and CVaR deviation measures in internal risk constraint, as suggested by Rockafellar and Uryasev (2002) and Rockafellar *et al.* (2006a, b). These functions measure downside risk as the negative deviation from the expected outcome, which corresponds to economical concepts of risk and is commonly applied in banks risk management, where risk typically is defined as a potential for adverse deviation from expected results (Jorion, 2000, p. 81).

*2.2.2 Modeling requirements on regulatory capital (Pillar 1).* According to the Pillar 1 of the Basel II Accord, banks must meet a total risk-based capital ratio of eligible capital and regulatory risk[6]. The total capital ratio is defined as the eligible regulatory capital divided by the risk-weighted assets. The total risk-weighted assets are determined by multiplying the capital requirements for market and operational risks by 12.5 (i.e. the reciprocal of the minimum capital ratio of 8 percent) and adding the resulting figures to the sum of risk-weighted assets for credit risk[7]. The capital ratio must be no lower than 8 percent[8].

The capital ratio is achieved by constraints on sub-portfolios of the bank book and the trading book and the risk is measured for these sub-portfolios[9]. The bank needs to meet minimum capital requirements for the credit risk of the bank book, for the overall operational risk, and for the market risk of the trading book[10]. The different risk types are limited by qualifying capital components of the regulatory capital[11]. Tier 1 capital, i.e. the “core” capital, includes common stockholders’ equity. Tier 1 basically represents ownership value, which serves as the primary cushion of losses if the bank faces financial difficulties. Tier 2 capital comprises supplementary capital, such as long-term subordinate debt and loan loss reserves. Tier 3 capital consists of senior

short-term debt. The maximum amounts of eligible Tiers 2 and 3 capitals are constrained with respect to the Tier 1 capital available.

Pillar 1 capital rules include several constraints. First, the credit and operational risks are limited with respect to the maximum amount of Tiers 1 and 2 capitals. Second, the market risk of trading book is limited by the unused components of Tiers 1 and 2 capitals from the first constraint plus eligible Tier 3 capital[12]. The use of Tier 3 capital is limited against unused reserves of Tiers 1 and 2 capital[13].

In optimization problems, we include constraints on different risk types according to regulatory rules. We also define constraints on the use of different capital reserves.

### 2.3 Optimization problem formulation

**2.3.1 Assumptions.** The Basel II Accord allows different possibilities to model the capital requirements in Pillar 1. For the implementation of Pillar 1 requirements, we make the following assumptions. For credit risk, we use the risk weights according to the standardized approach[14]. For market risk, we apply the internal VaR model[15]. Operational risk is considered with the basic indicator approach. We treat operational risk as a constant, as it is not linked to the decision variables of our optimization models. Banks may use regulatory capital reserves under Pillar 1 in a preference order to minimize funding costs on their capital reserves.

Under Pillar 2, we are using internal models based on bootstrapping of historical data for market and credit risk in the internal risk constraints[16]. For all other risks, we are assuming a constant capital buffer, which is not linked to the decision variables.

**2.3.2 Formulations of problem variations.** As we focus on approaches of risk integration and integrated risk measurement in the ICAAP, we formulate different optimization problems with respect to internal risk measurement in the economic capital constraints (Constraint Set 1). According to the different approaches of risk aggregation, which we have presented in Section 2.2, we achieve different problem variations.

At first, we consider the “worst-case approach”. In practice, capital limits for different risk types frequently are fixed in a simple top-down approach for different risk types and standalone risks are measured against these limits. By allowing variable limits of the different sub-portfolios, we demonstrate how capital can be used more efficiently by allowing variation of limits between sub-portfolio risk types, while risk of sub-portfolios is measured on a standalone basis and the overall capital use is not changed. In optimization two, we further include the hybrid approach and in optimization three, we consider the integrated approach. With respect to the integrated approach, we consider the special case where return distributions can be derived from historical data, thus avoiding additional assumptions on the inter-relations of different types of risk, which might otherwise bias the optimal asset allocations between market and credit assets[17].

Table I summarizes the different approaches of risk integration that we consider in the problems’ formulation.

**Table I.**  
Modeling the internal risk constraint (Constraint Set 1)

Problem 1: worst-case approach	Problem 2: hybrid approach	Problem 3: integrated approach
Worst-case aggregation with variable economic capital limits on market and credit portfolio	Correlation aggregation, economic capital on aggregated risk by correlation matrix	Integrated constraint, economic capital constraint on portfolio level

The considered optimization models differ only by the Constraint Set 1 for economic capital (ICAAP). Therefore, we formulate only optimization Problem 1, for other problems we provide only alternative Constraint Set 1. We consider in each Constraint Set 1 two cases:

- (1) VaR deviation; and
- (2) CVaR deviation for measuring risk.

Notations. We use the following notations in the optimization Problems 1-3 (Table II).

Optimization Problem 1 – worst-case approach. Maximize portfolio estimated return:

$$\max \sum_{i=1}^n r_i x_i \quad (4)$$

subject to:

- *Constraint Set 1 – economic capital (Pillar 2)*: Internal constraint on total economic capital:

$$\begin{aligned} \text{(a)} \quad & \sum_{j=1}^m \text{VaR\_Dev}_{\alpha\_int}(x_j) \leq C_{ec\_total\_risk} - C_{ec\_other\_risk} \\ \text{(b)} \quad & \sum_{j=1}^m \text{CVaR\_Dev}_{\alpha\_int}(x_j) \leq C_{ec\_total\_risk} - C_{ec\_other\_risk} \end{aligned} \quad (5)$$

- *Constraint Set 2 – regulatory capital (Pillar 1)*: Balance equation for the regulatory capital covering credit risk:

$$\sum_{i=1}^{n\_bb} w_i^{\text{reg\_cr}} x_i^{\text{bb}} + \sum_{i=1}^{n\_tb} w_i^{\text{reg\_sp}} x_i^{\text{tb}} + \text{reg\_op\_risk} = x_1^a + x_2^a. \quad (6)$$

Balance equation for the regulatory capital covering market risk:

$$12.5 * w^{\text{reg\_mult\_mr}} \text{VaR}_{99\text{percent}}(x^{\text{tb}}) \leq x_3^a + (C_{\text{tier}_1} - x_1^a) + (C_{\text{tier}_2} - x_2^a). \quad (7)$$

Constraint limiting unused Tier 2 and used Tier 3 capital vs unused Tier 1 capital:

$$x_3^a + (C_{\text{tier}_2} - x_2^a) \leq 2.5(C_{\text{tier}_1} - x_1^a). \quad (8)$$

Bounds on used Tiers 1-3 capital:

$$0 \leq x_i^a \leq C_{\text{tier}_k}, \quad i = 1, 2, 3. \quad (9)$$

Notation	Explanation
$n\_tb, n\_bb$	Number of assets in trading book and bank books, respectively
$n = n\_tb + n\_bb$	Total number of assets
$\mathbf{r} = (r_1, \dots, r_n)'$	Vector of estimated returns of assets
$\mathbf{x}^{bb} = \begin{pmatrix} x_1^{bb} \\ \dots \\ x_{n\_bb}^{bb} \end{pmatrix}$	(Sub-)vector of decision variables of the bank book
$\mathbf{x}^{tb} = \begin{pmatrix} x_1^{tb} \\ \dots \\ x_{n\_tb}^{tb} \end{pmatrix}$	(Sub-)vector of decision variables of the trading book
$\mathbf{x} = \begin{pmatrix} x_1 \\ \dots \\ x_{n\_bb+1} \\ \dots \\ x_n \end{pmatrix} = \begin{pmatrix} x_1^{bb} \\ \dots \\ x_{n\_bb}^{bb} \\ x_1^{tb} \\ \dots \\ x_{n\_tb}^{tb} \end{pmatrix} = \begin{pmatrix} \mathbf{x}^{bb} \\ \mathbf{x}^{tb} \end{pmatrix}$	Vector of decision variables, i.e. asset exposures
$\begin{pmatrix} 1 & \dots & \rho_{1m} \\ \dots & \dots & \dots \\ \rho_{m1} & \dots & 1 \end{pmatrix}$	Risk correlation matrix for sub-portfolios, $j = 1, \dots, m$
$C_{ec\_other\_risk}$	Available economic capital for other risk (Pillar 2)
$C_{ec\_total\_risk}$	Available economic capital for total risk (Pillar 2)
$C_{reg\_tier\_j}, i = 1, \dots, 3$	Available components of regulatory capital, tier $i$ , $i = 1, \dots, 3$
$reg\_op\_risk$	Constant for regulatory capital for operational risk (basic indicator approach)
$x_{ec\_cap}^j, j = 1, \dots, m$	Used economic capital in internal risk constraint for sub-portfolio $j$
$x_i^a, i = 1, \dots, 3$	Used regulatory tier $i$ , $i = 1, \dots, 3$ capital
$w_j^{reg\_cr}, j = 1, \dots, n\_bb$	Regulatory capital weights for credit assets of the bank book
$w_j^{reg\_sp}, j = 1, \dots, n\_bb$	Regulatory risk weights for market risk constraint: specific risk of assets
$w_{reg\_mult\_mr}$	Regulatory multiplication factor for VaR model
$\alpha\_int$	Confidence level for internal economic risk constraints (Pillar 2)
Total_Inv	Upper bound for overall investment exposures

**Table II.**  
Notation in optimization  
problems

- *Constraint Set 3 – exposure constraints*: Upper and lower bounds on decision variables (exposures):

$$\begin{aligned} l_i &\leq x_i^{\text{tb}} \leq u_i, & i = 1, \dots, n_{\text{tb}}, \\ l_i &\leq x_i^{\text{bb}} \leq u_i, & i = 1, \dots, n_{\text{bb}}. \end{aligned} \quad (10)$$

Constant investment amount over all assets:

$$\sum_{i=1}^{n_{\text{bb}}} x_i^{\text{bb}} + \sum_{i=1}^{n_{\text{tb}}} x_i^{\text{tb}} = \text{Total\_Inv}. \quad (11)$$

Further, we provide new variants of Constraint Set 1 on internal risk (Table I) and keep the same objective function and all other constraints in optimization Problems 2 and 3.

Optimization Problem 2 – hybrid approach:

- *Constraint Set 1 – economic capital (Pillar 2)*: Internal constraint on sub-portfolios  $x_j, j = 1, \dots, m$ :

$$\begin{aligned} \text{(a)} \quad \text{VaR\_Dev}_{\alpha_{\text{int}}}(x_j) &\leq x_{\text{ec\_cap}}^j, \\ \text{(b)} \quad \text{CVaR\_Dev}_{\alpha_{\text{int}}}(x_j) &\leq x_{\text{ec\_cap}}^j. \end{aligned} \quad (12)$$

Internal constraint on total economic capital:

$$\begin{aligned} &\sqrt{\begin{pmatrix} x_{\text{ec\_cap}}^1, \dots, x_{\text{ec\_cap}}^m \end{pmatrix} \begin{pmatrix} 1 & \dots & \rho_{1m} \\ \dots & \dots & \dots \\ \rho_{m1} & \dots & 1 \end{pmatrix} \begin{pmatrix} x_{\text{ec\_cap}}^1 \\ \dots \\ x_{\text{ec\_cap}}^m \end{pmatrix}} \\ &\leq C_{\text{ec\_total\_risk}} - C_{\text{ec\_other\_risk}} \end{aligned} \quad (13)$$

Optimization Problem 3 – integrated approach:

- *Constraint Set 1 – economic capital (Pillar 2)*: Internal constraint on total economic capital:

$$\begin{aligned} \text{(a)} \quad \text{VaR\_Dev}_{\alpha}(x) &\leq C_{\text{ec\_total\_risk}} - C_{\text{ec\_other\_risk}}, \\ \text{(b)} \quad \text{CVaR\_Dev}_{\alpha}(x) &\leq C_{\text{ec\_total\_risk}} - C_{\text{ec\_other\_risk}}. \end{aligned} \quad (14)$$

### 3. Case study

We used PSG to do the case studies. We posted MATLAB files to run these case studies in MATLAB environment on The MathWorks web site ([www.mathworks.com](http://www.mathworks.com)), in the file exchange-optimization area (search with the last name Serraino).

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### 3.1 Assumptions and setup

In our case study, we illustrate different risk measurement methods in the context of ICAAP requirements. We optimize models, as described in Section 2, to analyze effects of different methods of risk aggregation and measurement. Step 1 of the case study examines different risk aggregation approaches. Step 2 analyzes impact of different risk measures on aggregated risk measurement.

(a) *Portfolio assets.* We consider a typical bank book of a commercial bank. We use publicly available data (reports of the Federal Reserve Bank, web site: [www.federalreserve.gov/releases](http://www.federalreserve.gov/releases)) to determine the typical size and business assets of a large US commercial bank. We select four typical positions of bank assets: securities government bonds, corporate and industrial credit, real estate loans and Interbank loans[18]. We re-scale the exposures to a balance sheet exposure, which corresponds to the average balance sheet exposure of the five largest US commercial banks[19]. In addition, we define typical market assets. We assume an exposure of the trading book of 20 percent of total balance sheet exposure and split it equally into the USA and Euro market.

For building loss distributions, we bootstrap historical index returns to avoid additional modeling assumptions and to facilitate the analysis of portfolio effects for different approaches of internal risk aggregation. Table III summarizes the portfolio setup.

(b) *Assumptions on Constraint Set 1: economic capital.* Risk modeling in ICAAP is frequently related to a one-year time horizon, which corresponds to the accounting and capital planning periods. Accordingly, we choose a one-year holding period for all assets.

We use time series for market and credit assets from January 3, 2000 to December 26, 2007. In the first constraint, we apply a 99 percent confidence level for VaR deviation for the internal risk measurement.

(c) *Assumptions on Constraint Set 2: regulatory capital.* For the regulatory constraints, we apply approaches of the Basel rules for credit, market and operational risk as summarized in Section 2.2.2.

(d) *Further assumptions.* The upper bounds on available regulatory and economic capital are set equal to the corresponding capital use of the initial portfolio[20]. For the exposure constraints, we assume that exposures in the banking book can be reduced or increased by 20 percent, while for the trading book they can be reduced or increased by 100 percent of the initial values. Additionally, we are assuming that the total investment capital does not exceed the total investment capital of the initial portfolio.

### 3.2 Step 1 of case study

Our objective is to examine the effects of different approaches of risk aggregation in the context of integrated portfolio optimization under ICAAP. We solve Problems 1-3, as described in Section 2.3.2 with Constraint Set 1 (internal risk constraints), Constraint Set 2, and Constraint Set 3 (exposure constraints). We use VaR deviation (case a) in the internal risk constraint.

Our first hypothesis is that from Problems 1-3, we will observe higher risk-adjusted performance at the given level of economic capital, as portfolio effects are taken into consideration in a more risk-adjusted way: Problems 1 (standalone risk), Problems 2 (inter-correlation aggregation), and Problems 3 (integrated risk measurement).

	Exposure (billion US\$)	Indices	Rating	Regulatory credit risk weight	In percentage of initial exposure	
					Lower exposure bounds	Upper exposure bounds
<i>Setup credit portfolio</i>						
Securities government debt	220	Markit iBoxx \$ domestic sovereigns and sub-sovereigns AAA; total return index	AAA	0	-20	+20
Commercial and industrial credit	130	Markit iBoxx \$ domestic corporates BBB 1-3Y; total return index	BBB	100	-20	+20
Real estate loans	330	EPRA/NAREIT US index	BB (ass.)	35	-20	+20
Interbank loans	40	Markit iBoxx \$ Eurodollar financials AA 1-3Y; total return index	AA	20	-20	+20
<i>Setup market portfolio</i>						
Equity position USA	100	DJ industrial average		2	-100	+100
Equity position Europe	100	DJ EURO STOXX 50		2	-100	+100

**Table III.**  
Summary of portfolio setup and input data

We obtained the following optimal asset allocations in Problems 1-3 (Table IV).

In order to analyze risk-return ratios for optimal portfolios, we consider estimated returns and economic capital, which is allocated to cover the integrated risk with different approaches of risk aggregation. We define the portfolio risk-adjusted return on capital (RORAC) as the ratio of estimated return and economic capital of the portfolio (Table V)[21].

The optimized return is quite close for Portfolio 1 (\$20.66 billion) and Portfolio 2 (\$20.90 billion), while for Portfolio 3 (\$22.14 billion) it is higher. This results in RORAC increasing from 15.96 percent (in optimal solution 1) to 17.11 percent (in optimal solution 3); the optimal Portfolio 3 is the only one with higher return and RORAC than the initial portfolio. Thus, from the point of view of return adjusted for risk, the risk aggregation in Problem 3 allocated the economic capital in a more efficient way than the initial portfolio.

Summarizing, with our dataset we find evidence for our first hypothesis and conclude that the risk aggregation in ICAAP should be based on the risk-adjusted aggregation approach resulting in efficient use of economic capital. The diversification effects should be considered accurately.

### 3.3 Step 2 of case study

We consider the effect of using different risk measures in integrated risk assessment in the ICAAP. We follow Sarykalin *et al.* (2008) in stating that one should not compare VaR and CVaR with the same confidence level, as they measure different parts of the distribution. In the considered dataset distributions of instruments are not very skewed, therefore there exists a confidence level  $\alpha_1$  such that the optimization of

**Table IV.**  
Exposures of initial portfolio and optimal solutions in Step 1

Exposures (billion US\$)	Initial portfolio	Problem 1: worst-case approach	Problem 2: hybrid approach	Problem 3: integrated approach
$x_1^{bb}$ Government debt securities	220	264	264	229.91
$x_2^{bb}$ Commercial and industrial credit	130	146.52	131.86	104
$x_3^{bb}$ Real estate loans	330	299.67	311.83	396
$x_4^{bb}$ Interbank loans	40	48.00	48.00	32
$x_1^{fb}$ Equity position USA	100	161.81	164.31	158.09
$x_2^{fb}$ Equity position Europe	100	0	0	0
Total exposure	920	920	920	920

**Table V.**  
Risk-return ratios of initial portfolio and optimal solutions in Step 1

Portfolio risk-return ratios	Initial portfolio	Problem 1	Problem 2	Problem 3
Economic capital (billion US\$)	129.44	129.44	129.44	129.44
Estimated return (billion US\$)	21.70	20.66	20.90	22.14
RORAC = est. return/econ. capital (%)	16.76	15.96	16.15	17.11

$\alpha_1 - \text{VaR}$  is close to the optimization of  $\alpha_2 - \text{CVaR}$  in the sense that the objective values are close at optimality and the decision variables may have similar optimal positions. We find for the initial portfolio that:

$$\alpha_2 - \text{CVaR deviation} \approx \alpha_1 - \text{VaR deviation}$$

when  $\alpha_2 = 97.5$  percent and  $\alpha_1 = 99$  percent. Then we solve Problems 1-3 replacing 99 percent – VaR deviation in Constraint Set 1 with 97.5 percent – CVaR deviation. Our hypothesis is that the solutions of these three problems with 97.5 percent – CVaR deviation will be close to the solutions found with 99 percent – VaR deviation.

Tables VI and VII show the results of Step 2.

Comparison of Tables IV and V with Tables VI and VII show that the estimated return and RORAC with 99 percent – VaR deviation in Step 1 and with 97.5 percent – CVaR deviation in Step 2 are quite close for Problem 3, while for Problems 1 and 2 they are somewhat different and lead to a slightly higher RORAC for the CVaR deviation in Step 2. While we recognize that these results are dependent on the dataset, we point out that by using different values of confidence level  $\alpha$  in VaR and CVaR deviation it is possible to obtain optimal portfolios with similar properties. Thus, one should analyze properties of the dataset on which computations are based, with particular focus on the model for the tails of the distribution, before deciding to insert constraints on VaR or CVaR, as none of them is “better” than the other. With PSG it is possible to impose simultaneously both VaR and CVaR constraints. We then estimate and compare CVaR deviation of all optimal portfolios. Results are shown in Table VIII.

(1) Exposures (billion US\$)	(2) Initial portfolio	(3) Problem 1: worst-case approach	(4) Problem 2: hybrid approach	(5) Problem 3: integrated approach
$x_1^{\text{bb}}$ Securities				
government debt	220	264	264	227.73
$x_2^{\text{bb}}$ Commercial and industrial credit	130	133.43	119.74	104
$x_3^{\text{bb}}$ Real estate loans	330	396	321.88	391.17
$x_4^{\text{bb}}$ Interbank loans	40	48	48	32
$x_1^{\text{tb}}$ Equity position USA	100	78.57	166.37	165.10
$x_2^{\text{tb}}$ Equity position Europe	100	0	0	0
Total exposure	920	920	920	920

**Table VI.**  
Exposures of initial portfolio and optimal solutions with 97.5 percent – CVaR deviation in Step 2

Portfolio risk-return ratios	Initial portfolio	Problem 1	Problem 2	Problem 3
Economic capital (billion US\$)	129.44	129.44	129.44	129.44
Estimated return (billion US\$)	21.70	20.90	21.11	22.17
RORAC = est. return/econ. capital (%)	16.76	16.15	16.31	17.13

**Table VII.**  
Risk-return ratios of initial portfolio and optimal solutions with 97.5 percent – CVaR deviation in Step 2

In both Steps 1 and 2, the 95 percent CVaR deviation is higher for Problem 3 than for Problems 1 and 2, but in each solution it is lower than 95 percent CVaR of the initial portfolio. This is an expected result because the risk aggregation model in Problem 3 is the least conservative. In order to implement the risk strategy for the ongoing business, capital limits must be allocated top-down to the different business units. For the capital allocation of the optimal target portfolio, we suggest allocating capital according to the Euler allocation principle (Denault, 2001; Tasche, 1999). According to the Euler allocation principle, the risk contribution  $r_j(x)$  of the  $j$ th asset is based on the partial derivative of the overall risk measure  $r(x)$ :

$$r_j(x) = \frac{\partial r(x)}{\partial x_j} x_j, \quad j = 1, \dots, n. \quad (15)$$

PSG can calculate derivatives both for VaR and CVaR as well as many other risk measures. Capital allocations based on VaR-deviation for the optimal Portfolio 3 of Step 1 and for the initial portfolio are presented in Table IX.

We observe that risk contributions of assets 1, 2, and 4 are negative, which corresponds to a positive diversification effect of these assets in the portfolio. The main risk driver in the portfolio is the real estate loan position, which mirrors to some extent the actual credit crisis in financial markets. For the capital allocation, negative capital amounts lack meaningful interpretation. The issue of how to handle these effects of capital allocation is approached differently and not uniquely solved. The worst-case allocation sometimes is suggested as an alternative allocation method; however, this allocation approach is not efficient, as was discussed in Step 1 above. Summarizing, we conclude that the suggested optimization model, as described in Section 2 and applied

**Table VIII.**  
About 95 percent CVaR deviation of optimal portfolios

	Initial portfolio	Problem 1	Step 1 Problem 2	Problem 3	Problem 1	Step 2 Problem 2	Problem 3
95 percent – CVaR deviation	49.29	24.70	27.52	42.44	23.40	29.86	43.33

**Table IX.**  
Capital allocations based on VaR-deviation for the initial portfolio and the optimal Portfolio 3 in Step 1

	Euler allocation initial portfolio	Euler allocation optimal portfolio problem 3 (Step 1)
Securities government debt	-9.87	-8.90
Commercial and industrial credit	-1.71	-1.38
Real estate loans	74.16	99.29
Interbank loans	-0.80	-0.56
Equity position USA	25.32	32.24
Equity position Euro	34.54	0
Total capital <sup>a</sup>	121.64	120.69

**Note:** <sup>a</sup>The difference between total capital and VaR deviation of the portfolios (129.44) is due to approximation error in the derivatives calculation

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in this case study, supports a systematic generation of risk-return efficient target portfolios under the ICAAP. However, issues of practical implementation in risk aggregation and capital allocation still remain unsolved and require heuristic implementations.

#### 4. Conclusion

We suggested an optimization approach for a bank portfolio, which applies different methodologies of risk measurement in the context of actual regulatory requirements. We illustrated by an optimization example the generation of risk-return efficient portfolios with respect to Pillars 1 and 2 requirements. We analyzed the effects of different approaches of internal risk aggregation and suggested a systematic approach for risk strategy formulation based on risk-return optimized portfolios under the ICAAP. There is a need for further research and practical implementations especially for integrated risk measurement, risk relations modeling, and capital allocation. A future extension of this case study may be based on simulation-based scenarios for the sub-portfolios, especially for the credit portfolios. Different copula approaches, linking the marginal sub-portfolio distributions to the portfolio loss distribution, may be analyzed in this context. We suggest examining which of the copula approaches discussed in the literature are the most risk-return efficient in the use of economic capital, that is which copula approach allows to achieve maximum returns in the setting of the integrated optimization problem (Problem 3). Furthermore, the derivation of efficient capital allocation strategies for the optimized portfolios needs to be considered more in depth. Some practical issues of applying the Euler-allocation principle are not sufficiently investigated, in particular the treatment of capital allocation to assets with negative risk contributions.

#### Notes

1. Basel Committee on Banking Supervision (2004), Part 3: Pillar 2 – the supervisory review process, § 725.
2. For a preceding problem formulation refer to Theiler (2004).
3. See Rosenberg and Schuermann (2006) for a review on related literature on integrated risk modeling.
4. The copula couples the marginal distributions together to form a joint distribution. The dependence relationship is entirely determined by the copula, while scaling and shape (mean, standard deviation, skewness, and kurtosis) are entirely determined by the marginals (Rosenberg and Schuermann, 2006, p. 13).
5. CVaR is a coherent risk measure in the sense of Artzner *et al.* (1997, 1999), see proof in Rockafellar and Uryasev (2002) and a coherent deviation measure in the sense of Rockafellar *et al.* (2006a, b). For a discussion of the appropriateness of CVaR, also refer to Acerbi and Tasche (2002) and Bertsimas *et al.* (2004), for a comparison of VaR and CVaR to Yoshida and Yamai (2001a, b).
6. See Basel Committee on Banking Supervision (1996), Introduction II (b), p. 8. For an explanation, refer also to US Accounting Office (1998, p. 123).
7. Basel Committee on Banking Supervision (2004), Part 2 I B 44.
8. Basel Committee on Banking Supervision (2004), Part 2 I 40.

9. The bank book comprises all “non-trading” assets, while the trading book comprises all positions the bank is holding for short term trading purposes. See Basel Committee on Banking Supervision (1996), Introduction I (a), Paragraph 2.
10. Only banks with a significant market risk exposure are required to calculate a risk-based capital ratio that takes into account market risk in addition to credit and operational risk.
11. For definition refer to Basel Committee on Banking Supervision (1996), Introduction II (a), p. 7. For an explanation, see also US Accounting Office (1998, pp. 119 and 122).
12. Basel Committee on Banking Supervision (1988), Part I; for a summary, see, also US Accounting Office (1998, pp. 116-127).
13. Basel Committee on Banking Supervision (1988), Part II (a) 1.
14. Basel Committee on Banking Supervision (2004), Part II, 2.
15. Basel Committee on Banking Supervision (2004), Part VI and Basel Committee on Banking Supervision (1996).
16. We are using one-year log returns for both market and credit assets and a 99 percent confidence interval for VaR deviation. Refer also to Section 3.
17. This study should further be extended by an inclusion of copula approaches for integrated risk measurement.
18. Form H.8 (510): Assets and Liabilities of Commercial Banks in the USA of December 2007. We omitted consumer credits, as we lacked sufficient data input for the historical index returns.
19. Refer to the web site: [www.federalreserve.gov/releases](http://www.federalreserve.gov/releases) data of December 31, 2007, scaled up to the next ten billions; Largest Banks List: Insured US-Chartered Commercial Banks that have consolidated assets of \$300 million or more, ranked by consolidated assets as of December 31, 2007.
20. We assume that the bank only uses Tiers 1 and 2 capital. We assume all other risks under Pillars 1 and 2, which are treated as constants in the optimization model, equal to zero.
21. In the context of this analysis, we abstract from considering further cost components in the nominator of RORAC. For practical implementation, the expected return should be adjusted especially for transaction cost, expected, and unexpected losses, i.e. capital costs.

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**Corresponding author**

Stan Uryasev can be contacted at: [uryasev@ufl.edu](mailto:uryasev@ufl.edu)