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Risk Return Management Approach for the Bank Portfolio

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ABSTRACT

In a competitive environment banks are forced to implement an enterprise-wide integrated risk return management. We give a survey of a risk return management approach for the bank portfolio that generates basic decision information for an integrated risk return management (Theiler, 2002). We formulate an optimization model that maximizes the expected returns subject to internal and regulatory loss risk limits. The internal risk constraint is based on the risk measure of conditional value-at-risk (CVaR), that has proven to be appropriate for measuring enterprise-wide loss risk (Acerbi and Tasche, 2002; Artzner *et al*., 1999; Rockafellar and Uryasev, 2002). The regulatory risk constraints represent the "Basel Rules" of loss risk limitation. We consider the present "Basel I" Capital Accord (Basel, 1988, 1996) as well as the new "Basel II" Capital Accord (Basel, 2003). We apply an algorithm by Rockafellar and Uryasev (2000) to solve the optimization problem. In order to translate the optimum portfolio into operational targets, we derive risk return keys for the business lines from the optimum solution (Denault, 2001; Patrik *et al*., 1999; Tasche, 1999). Finally we illustrate the risk return management approach by a practical example.

19.1 INTRODUCTION

In a competitive environment banks are forced to implement an enterprise-wide integrated risk return management. Bank management decisions need to take into account increasing risks in times of decreasing returns. Financial risks have to be limited and managed from a enterprise-wide portfolio perspective, where risk limitation rules must be accomplished from internal and regulatory points of view. Expected returns need to be maximized subject to these constraints and banks must invest scarce resources in business activities yielding highest risk adjusted performance ratios.

We introduce a risk return management approach (RRM Approach) for the bank portfolio that generates basic management information for an enterprise-wide integrated risk return management. Our contribution is to apply recently developed methods of risk measurement, capital allocation and portfolio optimization to the bank portfolio in order to develop instruments for an enterprise-wide risk return management. The RRM Approach determines optimal bank portfolios and derives efficient risk return objectives for the business lines, thus providing consistent planning information on where to invest the scarce capital resources in order to maximize risk adjusted performance of the bank portfolio.

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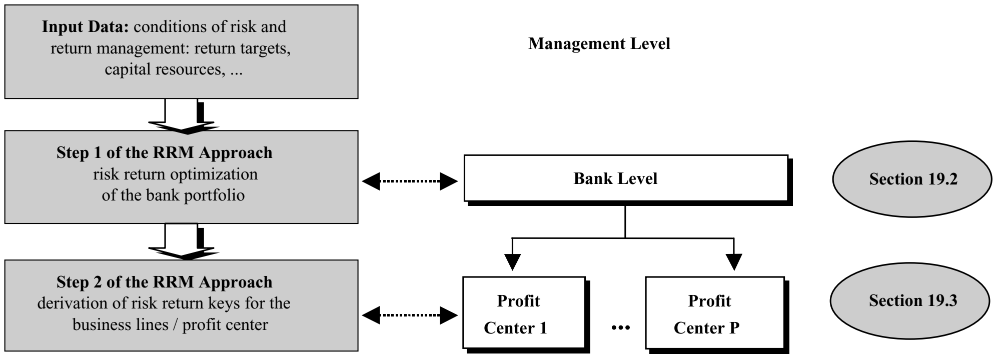

Figure 19.1 Survey on the risk return management approach for the bank portfolio

The RRM Approach consists of two steps and proceeds top down from the bank level to the business lines, that are also denoted as profit center. The first step consists of an optimization model on the bank level that maximizes the expected returns of the bank portfolio subject to enterprise-wide loss risk limitations from the internal and the regulatory perspective. The internal risk constraint is based on the new risk measure of conditional value-at-risk (CVaR), that has been proved to be appropriate for measuring enterprise-wide loss risk.[1] The first step of the RRM Approach applies the algorithm by Rockafellar and Uryasev (2000) to solve the optimization problem and takes into account the regulatory "Basel rules" of risk limitation and considers the present as well as the new Basel Capital Accord.[2] The second step of the RRM Approach derives operational risk return objectives for the business lines from the optimum portfolio. It aggregates the risk return ratios on the business line level as basic target ratios for the enterprise-wide risk return management.[3]

To develop the RRM Approach, we proceed as follows. In section 19.2 we introduce the optimization model that is in the focus of the step 1 of the RRM Approach. We identify the conditional value-at-risk as an appropriate risk measure for the internal risk measurement. We describe the optimization algorithm for the solution of the basic optimization problem to maximize expected returns with respect to the CVaR risk constraint. We extend the problem to an optimization model for the bank portfolio by integrating the regulatory loss risk constraints. We develop the optimization model with respect to the present "Basel I" Capital Accord and integrate the rules of the new Accord of "Basel II". In section 19.3 we describe step 2 of the RRM Approach that derives risk return ratios for the business lines from the optimal portfolio of step 1 of the RRM Approach. We first achieve the risk and return contributions of the assets and then aggregate these on the business line level. The impact of the risk return management approach is shown by an application example in section 19.4. We conclude by a summary and outlook in section 19.5.

Figure 19.1 summarizes the basic structure of the RRM Approach.

[1] We refer to Acerbi and Tasche (2002), Artzner *et al*. (1999) and Rockafellar and Uryasev (2002).
[2] By the term of the "Basel rules" we refer to the regulatory rules by the Basel Committee of Banking Supervision (BCBS). We consider the present Capital Accord of 1988, also refered to as "Basel I" (see Basel, 1988), in combination with its Amendments to Incorporate Market Risks (see Basel, 1996), as well as the New Capital Accord, also referred to as "Basel II" (see Basel, 2003).
[3] Refer to Theiler (2002).

19.2 STEP 1 OF THE RRM APPROACH: OPTIMIZATION MODEL FOR THE BANK PORTFOLIO

19.2.1 Survey

In its planning process the bank needs to define a risk return efficient plan portfolio for the next business year. The plan portfolio should yield maximal expected returns and meet the enterprise-wide risk constraints from the internal and the regulatory point of view. From an internal perspective, the bank limits its loss risk by the economic capital, i.e. equity-related capital resources, that the bank applies to cover occurring losses.[4] At the same time, the bank must observe legal loss risk boundaries, given by the "Basel rules", which constrain the loss risks from the regulatory perspective and specify capital elements to cover the different kinds of risk.

In step 1 of the RRM Approach we formulate an optimization model that determines risk return efficient portfolios. We assume a planning horizon of one year. We formulate a one-period optimization model that maximizes the expected returns of the bank portfolio with respect to internal and regulatory loss risk limits to the one-year horizon. The exposures of the assets represent the decision variables.[5] We achieve the following general model structure of the optimization problem for the bank portfolio:

objective function: maximize expected returns subject to constraints
constraint 1: internal risk $\leq$ economic capital
constraint 2: (a) regulatory risk $\leq$ regulatory capital (19.1)
(b) regulatory constraints on the regulatory capital components
constraint 3: position bounds, definition of the feasible solutions

We develop the optimization model in different steps. First we formulate the basic optimization problem of maximizing expected returns with respect to the internal risk constraint (section 19.2.2). We define a risk measure that is appropriate to measure the internal loss risk of the bank portfolio (section 19.2.2.1) and introduce an optimization algorithm for the solution of the basic risk return optimization problem of maximizing expected returns with respect to the internal risk constraint (section 19.2.2.2). In the section 19.2.3 we integrate the regulatory risk constraint of the "Basel" rules into the optimization model. We first consider the present Capital Accord (section 19.2.3.1) and then outline how the new rules of the "Basel II" Accord are integrated into the optimization model (section 19.2.3.2).

19.2.2 Modeling the internal risk constraint

19.2.2.1 Definition of the internal risk measure for the bank portfolio

We first determine properties that a risk measure for the bank portfolio should meet. We then assess alternative measures of risk and choose an appropriate risk measure that meets the following *requirements*:

[4] The economic capital often is defined as a subset of the bank's equity. Where national law allows the accumulation of hidden reserves, these are commonly applied as elements of the economic capital.

[5] For the long-term perspective of the one-year planning horizon it suffices to take into account *aggregate positions*; e.g. depending on the organizational structure of the bank, one may consider product or customer segments.

(1) Whereas in finance theory risk was commonly defined as the variability of return,[6] in modern risk management practice risk has come to mean "danger of loss" instead of symmetric deviation from an expectation, thus accounting for *negative* outcomes, if returns are not symmetrically distributed.[7] In its risk management processes, the bank management must limit the risk of missing expected and publicly announced return targets to maintain the soundness and credit worthiness of the bank. Therefore, we define loss risk in the RRM Approach as the *danger of negative deviation of the uncertain portfolio value from the expected portfolio value at the horizon*.

We use the following notation. Let $x = (x_1, \ldots, x_n)'$ be the vector of the positions of the bank assets and $y = (y_1, \ldots, y_n)'$ the vector of the corresponding market prices. According to the economic perspective of risk as the danger of missing a planned target, we define the portfolio loss function $L(x, y)$ as the difference of the uncertain from the expected portfolio value at the horizon:[8]

$$L(x, y) = E[\mathbf{y}]'\mathbf{x} - \mathbf{y}'\mathbf{x} = \sum_{i=1}^{n} E[y_i]x_i - y_i x_i \tag{19.2}$$

The risk measure we apply should *capture large deviations, i.e. extreme losses occurring in the tail of the corresponding loss risk distribution*.

(2) The risk measure should take into account the management's risk attitude that typically is specified by the *confidence level* applied in the enterprise-wide risk management, that frequently is related to the bank's solvency probability.

(3) There are different kinds of risk inherent in the bank portfolio. Risk managers develop models to quantify credit, market and operational risk. The risk measure should be suitable to measure loss risk from any kind of loss distribution and especially *allow an integrated risk measurement of bank portfolio-wide losses*.

(4) By the concept of the "coherent measures of risk" Artzner *et al*. (1999) have developed an axiomatic framework of desirable properties that a risk measure should meet. From a point of view of a supervising authority, *the risk measure should allow the distinction of acceptable and not acceptable portfolios with respect to their risk*. Any risk measure that meets the four axioms of coherence, as there are the subadditivity, homogeneity, monotonicity and translation invariance, is an appropriate risk measure in this framework.

Artzner *et al*. (1999) base their analysis on the definition of loss as negative outcomes. According to requirement (1) we refer to loss as the negative deviation of the uncertain value from an expected value. Rockafellar *et al*. (2002) have extended the concept of coherent risk measures to the concept of *coherent deviation measures*, a parallel class of functionals that quantifies discrepancies of random outcomes from an expected value. The authors establish a one-to-one correspondence between risk and deviation measures. In accordance with our definition of loss risk we demand the risk measure applied in the RRM Approach to *meet the axioms of a coherent deviation measure in the sense of Rockafellar et al. (2002)*.[9]

(5) For the support of an efficient risk return management of the overall bank portfolio, the risk measure should allow the determination of risk return optimal portfolios and be easily implemented in a risk return optimization algorithm for the bank portfolio.

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[6] See, for example, a survey in Crouhy *et al*. (2001, p. 23).
[7] See, for instance, Bueschgen (1998, p. 865), Jorion (2000, p. 81) or Rockafellar *et al*. (2002, p. 2).
[8] See, for instance, Jorion (2000, pp. 81 and 109).
[9] See Definition 1, General Deviation Measures, Rockafellar *et al*. (2002, p. 3).

(6) Expected utility maximization is a commonly accepted preference representation in finance. As the risk measure applied in the RRM Approach should ensure rational risk management decisions, we demand the risk measure applied in the RRM Approach to be consistent with expected utility maximization.[10]
(7) Furthermore, the risk measure should be practicable and understandable for a successful implementation in practice.

We take into account alternative risk measures commonly applied in finance with respect to the above requirements, as there are variance and higher central moments, lower partial moments and quantile-based risk measures. With respect to the first requirement, we observe that risk measures based on the central moment, as the variance and higher central moments, capture risk around the expectation of the loss distribution. As they do not focus on tail risk, they do not meet requirement (1) and therefore are excluded from further consideration. Analyzing different tail risk measures, we notice that the lower partial moments measure risk with respect to a constant hurdle rate, rather than taking into account a specified enterprise-wide confidence level of risk management. Therefore, the risk measures of lower partial moments are ruled out from further consideration, as they do not meet requirement (2).[11]

We focus our attention on quantile-based risk measures and consider the value-at-risk and the conditional value-at-risk, that frequently also is referred to as "expected shortfall". We base the definitions on the above loss definition of equation (19.2). According to the idea of coherent deviation measures by Rockafellar *et al*. (2002), we define the *value-at-risk deviation* $VaR(L(x, y))$ as the α-quantile of the portfolio loss distribution:

$$VaR_\alpha(L(x, y)) = \inf\{z \in \mathcal{R} \mid P(L(x, y) \leq z) \geq \alpha\} \tag{19.3}$$

We define the corresponding *conditional value-at-risk deviation* $CVaR_\alpha(L(x, y))$ of the portfolio loss as the conditional expectation beyond the corresponding VaR:[12]

$$CVaR_\alpha(L(x, y)) = E[L(x, y) \mid L(x, y) \geq VaR_\alpha(L(x, y))] \tag{19.4}$$

Figure 19.2 illustrates a hypothetical loss distribution of the bank portfolio and the deviation measures of VaR and CVaR.

Value-at-risk, commonly applied in finance for market risk measurement, may lack the elementary property of subadditivity, if the loss distributions are not normal and is not coherent in this case.[13] It has been proved that CVaR is a coherent risk and deviation measure.[14] Rockafellar/Uryasev (2000) show that is a convex measure and ensures the existence of a risk minimum portfolio on a convex set and the solvability of the optimization problem (P) and thus satisfies the properties (4) and (5). Rockafellar and Uryasev (2002) point out that it is appropriate to measure loss risk from any kind of distribution, also with discrete probabilities, and therefore as well meets the requirement (3).

Yamai and Yoshiba (2001b) have shown that CVaR is consistent with expected utility maximization by risk-averse investors under more lenient conditions than VaR. The authors prove

[10] A risk measure is said to be *consistent with expected utility maximization*, if it yields the same ranking of portfolios. See Yoshiba and Yamai (2001b, p. 3).

[11] Furthermore, it can be shown that the lower partial moments in general do not meet the coherence property as claimed by requirement (4).

[12] For a detailed definition, refer to Rockafellar and Uryasev (2002). In the case of discontinuities at the α-quantile, CVaR should be defined as a weighted average of VaR and the conditional expectation beyond VaR. See Acerbi and Tasche (2002, proposition 4.1, equation (4.4)), and Rockafellar and Uryasev (2002, proposition 8).

[13] See, for example, Artzner *et al*. (1999).

[14] Refer to Artzner *et al*. (1999) and Rockafellar and Uryasev (2002).

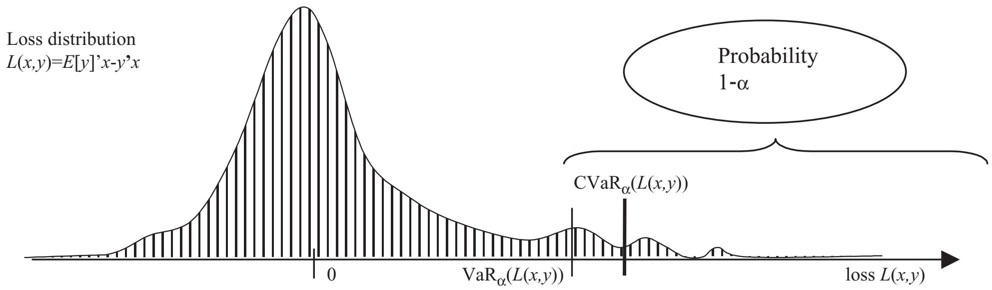

Figure 19.2 Loss distribution $L(x, y)$ and deviation measures VaR and CVaR

that CVaR is consistent with expected utility maximization if portfolios are ranked by second-order stochastic dominance.[15] On the other hand they show that this is true for VaR under the additional assumption that the losses of the portfolios have an elliptical distribution with finite variance and the same mean.[16] We conclude that CVaR meets the property (6) in a better way than VaR does.[17]

Furthermore, CVaR can be estimated from a given sample as the mean of the losses in the tail of the distribution beyond VaR. However, Yamai and Yoshiba (2001b) point out that the reliability of expected shortfall depends on the stability of estimation and the choice of efficient back-testing methods.[18] As the CVaR estimation is based on the corresponding VaR and requires only one further step of calculation once VaR is determined from a given sample, we are confident that CVaR may find acceptance in practice, despite the intensive discussions expected along with the introduction of a new risk measure.[19] Summarizing, we conclude that CVaR meets the above requirements of a risk measure and we therefore base the loss risk measurement in the RRM Approach on CVaR.

19.2.2.2 Formulation of the basic risk return optimization problem with respect to the internal risk constraint

We consider the basic optimization problem (P_{CVaR}) to maximize the expected returns with respect to a CVaR risk constraint. We apply an algorithm by Rockafellar and Uryasev (2000) to solve this optimization problem and achieve a linear optimization model. In brief, this approach makes use of the fact that CVaR is expressed in terms of a convex auxiliary function that is approximated by a piecewise linear function. Based on a scenario generation of the market

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[15] Yoshiba and Yamai (2001b, Theorem 15, p. 21).

[16] Yoshiba and Yamai (2001b, Theorem 14, p. 20).

[17] Yoshiba and Yamai point out that CVaR is *not* consitent with expected utility maximization if portfolios are *not* ranked by second-order stochastic dominace. Alternatively, they suggest the use of lower partial moments of second or higher order (see Yoshiba and Yamai (2001b, p. 23)), which, however, in turn lack the above required properties (2) and (4).

[18] Yoshiba and Yamai point out that asset price correlations observed in normal conditions may break down in extreme market situations. This correlation breakdown makes it impossible to estimate the tail of the portfolio loss distribution properly with conventional Monte Carlo simulations that apply constant correlations. Another problem in the application of CVaR is, that the back-testing using CVaR needs to compare the average of realized losses beyond the VaR level. This requires more data than the back-testing using VaR. See Yoshiba and Yamai (2001a, pp. 80–81). On the other hand Kerkhof and Melenber point out that tests for expected shortfall have better performance than tests for VaR in realistic financial sample sizes. See Kerkhof and Melenberg (2003).

[19] Often it is opposed, that the use of CVaR creates higher costs, as CVaR requires a higher amount of economic capital allocated to cover risks than VaR does. One may suggest that the definition of CVaR does not require such high confidence levels to account for extreme losses, e.g. 99.5% to 99.9%, as VaR. CVaR takes into account extreme tail risks of the distribution also at a lower confidence level. Reducing the confidence level would require less economic capital and also allow the use of an integrated confidence level, for example 99.0%, for the different kinds of risk, especially for an integrated market and credit risk measurement.

prices of the portfolio assets, the initial CVaR constraint is replaced by a set of linear constraints and the solution of the initial CVaR-optimization problem is approximated by a solution of a linear optimization model.

Let $x = (x_1, \ldots, x_n)'$ be the decision variable, i.e. the positions of the portfolio assets, where n denotes the total number of assets, and $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)'$ the vector of the expected returns of the assets. We define a linear objective function $\mu(x)$ of the expected portfolio return, that is maximized in the optimization problem:

$$\mu(x) = \boldsymbol{\mu}'x = \sum_{j=1}^{n} \mu_{\mathrm{j}} x_{\mathrm{j}} \tag{19.5}$$

The maximum amount of economic capital that the bank applies to cover internal losses is denoted as *ec_cap_max*. It represents the upper bound of portfolio-wide loss risk, that is measured by CVaR. Let α denote the enterprise-wide confidence level of internal risk measurement, $y_1, \ldots, y_K$ the set of market prices of the portfolio assets, where K denotes the total number of scenarios, $\mathrm{z}_1, \ldots, \mathrm{z}_K$ a set of K nonnegative auxiliary variables and q a real number. The CVaR constraint is modeled by the following set of inequalities:

$$\begin{array}{lll}
\text{(i)} & q + \frac{1}{1-\alpha} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \leq ec_cap_max & \left.\right\} \begin{array}{l}\text{In the optimum solution} \\ \text{— } q \text{ is an estimate of the portfolio VaR,} \\ \text{—the left-hand side of (i) is an estimate} \\ \quad \text{of the portfolio CVaR.}\end{array} \\
\text{(ii)} & L(x, y_k) - q \leq z_k, \qquad k = 1, \ldots, K & \\
\text{(iii)} & -z_k \leq 0, \qquad k = 1, \ldots, K & \\
\text{(iv)} & q \in \mathcal{R} &
\end{array} \tag{19.6}$$

Rockafellar and Uryasev (2000) show that in the optimal solution the left-hand side of the first constraint (i) provides an estimate of the portfolio CVaR. In the optimum portfolio, the free variable q approximates the portfolio VaR at the given confidence level α. In the left-hand term of the constraint (i) the mean of the auxiliary variables z_1 to z_K will count for the mean of the losses exceeding q and is scaled up by the tail probability. Thus the left-hand side of the equation (19.6)(i) provides an estimate for the portfolio CVaR in the optimal solution.

In order to define the area of the feasible solutions, we additionally define vectors *up_bound* and *low_bound* of upper and lower exposure bounds of the portfolio assets:

$$\begin{array}{ll}
\text{(i)} & up_bound = (up_bound_1, \ldots, up_bound_n)' \\
\text{(ii)} & low_bound = (low_bound_1, \ldots, low_bound_n)'
\end{array} \tag{19.7}$$

We constraint the exposures by these box constraints:

$$low\_bound \leq x \leq up\_bound \tag{19.8}$$

i.e. $low\_bound_i \leq x_i \leq up\_bound_i, \; i = 1, \ldots, n.$

We summarize the basic optimization model (P_{CVaR}) as follows:

Optimization model (P_{CVaR})

(i) $$\mu(x) = \boldsymbol{\mu}'x = \sum_{i=1}^{n} \mu_i x_i \quad \rightarrow max$$

Constraint 1: Internal risk constraint (19.9)

$$\text{(ii)} \quad q + \frac{1}{1-\alpha} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \leq ec\_cap\_max$$

$$\text{(iii)} \quad L(x, y_k) - q \leq z_k, \qquad k = 1, \ldots, K$$

$$\text{(iv)} \quad -z_k \leq 0, \qquad k = 1, \ldots, K$$

$$\text{(v)} \quad q \in \mathcal{R}$$

Constraint 3: Boundaries of the feasible solutions

$$\text{(vi)} \quad low\_bound \leq x \leq up\_bound$$

The problem (P_{CVaR}) represents a linear optimization model. Rockafellar and Uryasev (2000) point out that this optimization approach can be solved very efficiently by using linear optimization techniques.

19.2.3 Integration of the regulatory risk constraint into the optimization model

In this section we outline how the regulatory risk constraints of the "Basel" rules are integrated into the basic optimization model (P_{CVaR}). First we formulate the optimization problem based on the present "Basel I" Capital Accord (section 19.2.3.1). We then outline how the new rules of the "Basel II" Capital Accord are integrated into the optimization model (section 19.2.3.2).

19.2.3.1 Formulation of the optimization model with respect to the "Basel I" Capital Accord

According to the present "Basel I" Accord,[20] banks must meet the following risk-based capital ratio of eligible capital and the regulatory risk,[21] in order to limit the risk in relation to the capital available to cover risks:

$$\frac{\text{tier 1 capital} + \text{tier 2 capital} + \text{tier 3 capital}}{\text{credit risk of the bank book} + 12.5 * \text{market risk of the trading book}} \geq 8.00\% \qquad (19.10)$$

Regulations distinguish the bank book and the trading book positions of the bank portfolio.[22] The bank needs to meet minimum capital requirements for the credit risk of the bank book and for the market risk of the trading book. The different kinds of risks are limited by qualifying capital components of the regulatory capital.[23] The tier 1 capital, the "core" capital, includes common stock-holders' equity. Tier 1 basically represents ownership value, that serves as primary cushion of losses, if the bank faces financial difficulties. The tier 2 capital comprises supplementary capital, such as long-term subordinate debt and loan loss reserves. Tier 3 capital consists of senior short-term debt. The maximum amounts of eligible tier 2 and tier 3 capital are constrained with respect to tier 1 capital available.

The minimum capital ratio ensures that the bank's regulatory risk does not exceed the amount of regulatory capital eligible to cover the loss risks. This loss risk rule is composed of two steps. In a first constraint, the credit risk of the bank book is limited with respect to the maximum

[20] Refer to Basel (1988) and Basel (1996).
[21] See Basel (1996), Introduction II (b), p. 8. For an explanation, refer also to United States Accounting Office (1998, p. 123).
[22] The bank book comprises all "non-trading" assets, while the trading book comprises all positions the bank is intentionally holding for short-term trading purposes. See Basel (1996), Introduction I (a), paragraph 2.
[23] For definition refer to Basel (1996), Introduction II (a), p. 7. For an explanation, refer also to United States Accounting Office (1998, pp. 119 and 122).

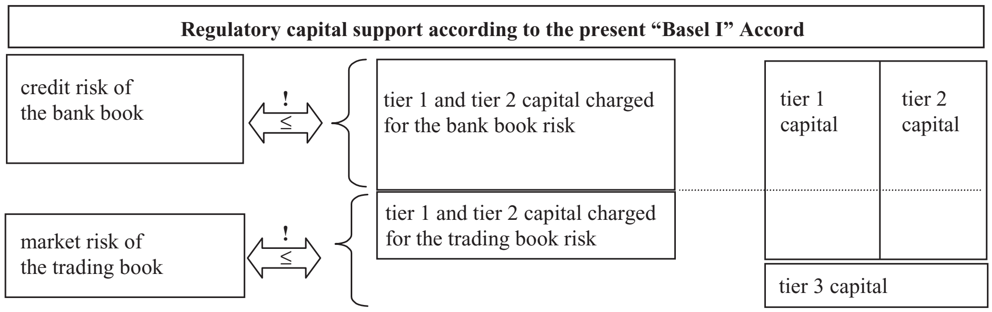

Figure 19.3 Regulatory capital support according to the present "Basel I" Accord

amount of tier 1 and tier 2 capital. Secondly, the market risks of the trading book are limited by the unused components of the tier 1 and tier 2 capital of the first constraint plus the eligible tier 3 capital. Figure 19.3 gives a survey on the basic structure of the regulatory loss risk limitation according to the present Capital Accord.

In the following we describe how capital is charged for the different kinds of risk. First we distinguish if the portfolio positions are attributed to the bank book or the trading book. We define sub-vectors of the portfolio vector $x = (x_1, \ldots, x_n)'$. Let $x^{bb} = (x_1^{bb}, \ldots, x_{n\_bb}^{bb})'$ denote the sub-vector of the bank book positions and $x^{tb} = (x_1^{tb}, \ldots, x_{n\_tb}^{tb})'$ the sub-vector of the trading book assets, where n_bb and n_tb denote the number of assets attributed to the bank book and the trading book, respectively, such that $n\_bb + n\_tb = n$. We may consider the following order of the portfolio assets:

$$x = \begin{pmatrix} x_1 \\ \cdots \\ \\ \\ \cdots \\ x_n \end{pmatrix} = \begin{pmatrix} x_1^{bb} \\ \cdots \\ x_{n\_bb}^{bb} \\ x_1^{tb} \\ \cdots \\ x_{n\_tb}^{tb} \end{pmatrix} = \begin{pmatrix} x^{bb} \\ \\ x^{tb} \end{pmatrix} \begin{matrix} \left.\right\} \text{Vector of the bank book assets} \\ \left.\right\} \text{Vector of the trading book assets} \end{matrix} \qquad (19.11)$$

The regulatory capital for the *credit risk of the bank book assets* is charged by linear coefficients. According to the rules of "Basel I", the exposure of every asset is transformed into a credit equivalent. Its exposure is multiplied by a credit conversion factor and a risk weight according to the type of borrower.[24] Let $reg\_cap = (reg\_cap_1, \ldots, reg\_cap_{n\_bb})'$ denote the resulting vector of the capital charge coefficients for the bank book assets, then the capital charge $cap\_charge^{bb}(x^{bb})$ of the bank book is equal to

$$cap\_charge^{bb}(x^{bb}) = reg\_cap' x^{bb} = \sum_{j=1}^{n\_bb} reg\_cap_j x_j^{bb} \qquad (19.12)$$

The capital charge of the sum of the risk weighted assets is limited by the regulatory capital resources of tier 1 plus tier 2 capital available, as illustrated by the first constraint in Figure 19.3. Let *tier_1_bb* and *tier_2_bb* denote the tier 1 and tier 2 capital charged for the bank book

[24] See Basel (1988), section II, paragraphs 28 to 43.

risk, then

$$cap\_charge^{bb}(x^{bb}) = tier\_1\_bb + tier\_2\_bb \leq tier\_1\_max + tier\_2\_max^{25} \quad (19.13)$$

The capital charge for *the trading book* comprises the general market risk and the issuer-specific risk of the trading book positions.

$$cap\_charge^{tb}(x^{tb}) = spec\_risk(x^{tb}) + gen\_market\_risk(x^{tb}) \quad (19.14)$$

The capital requirements for the general market risk are intended to cover loss risks arising from changes in the underlying market rates. The capital charge for the specific risk is supposed to protect against an adverse movement in the price of an individual security owing to factors related to the individual issuer.[26] We assume that the bank uses the standard approach to measure the specific risk[27] and an internal market risk model to estimate the general market risk of the trading book positions.[28]

According to the standard approach, we model linear constraints for the specific risk. Let $spec\_risk = (spec\_risk_1, \ldots, spec\_risk_{n\_tb})'$ denote the vector of the specific risk weights of the trading book assets. We achieve the specific risk $spec\_risk(x^{tb})$ of the trading book as

$$\text{(i)} \qquad spec\_risk(\boldsymbol{x}^{tb}) = spec\_risk'\boldsymbol{x}^{tb} = \sum_{j=1}^{n\_tb} spec\_risk_j x_j^{tb} \quad (19.15)$$

We assume that the *general market risk of the trading book* is measured by the VaR of the trading book, as estimated by an internal market risk model using the regulatory parameters.[29] We apply the regulatory confidence level of 99%. As we generate planning information to the one-year planning horizon, we assume a time horizon of one year instead of the 10-day liquidation period of the "Basel" rules.[30]

Let $y^{tb} = (y_1^{tb}, \ldots, y_{n\_tb}^{tb})'$ denote the vector of the market prices of the trading book assets. We define the loss function $L^{tb}(x^{tb}, y^{tb})$ for the trading book as follows:[31]

$$L^{tb}(\boldsymbol{x}^{tb}, \boldsymbol{y}^{tb}) = E[y^{tb}]'x^{tb} - y^{tb\prime}x^{tb} \quad (19.16)$$

Let $VaR_{99\%}^{tb}(L^{tb}(y^{tb}, \boldsymbol{x}^{tb}))$ denote the 99%-VaR of the general market risk of the trading book, then

$$gen\_market\_risk(x^{tb}) = mult \cdot VaR_{99\%}^{tb}(L^{tb}(x^{tb}, y^{tb})) \quad (19.17)$$

where *mult* is the regulatory multiplication factor of the internal model that depends on the forecast quality of the model applied.[32]

[25] In the optimization model we may apply a penalty function in the goal function that forces the tier 2 capital to be used first, in order to keep free capacity of the tier 1 capital for the market risk capital charge; see equation (19.22) below.

[26] See Basel (1996), Part A, paragraph A.1 and A.2.

[27] See Basel (1996), Part A.

[28] See Basel (1996), Part B.

[29] These are defined in Basel (1996), Part B.

[30] Modeling the market risk to the one-year horizon may need research, how to properly model the short-term risks in the context of the one-year horizon.

[31] As we are assuming a liquidation period of one-year, we take into account the deviation of the uncertain value from the expected value of the trading book. In the context of a short-term analysis, e.g. for a liquidation period of 10 days, one may consider the expectation $E[\mathbf{y^{tb}}]$ to be equal to zero. For alternative definitions of the loss function and the corresponding risk measures, see for example, Rockafellar *et al.* (2002) or Jorion (2000, p. 109).

[32] The multiplying factor is set by supervisory authorities on the basis of their assessment of the quality of the bank's risk management system. See Basel (1996), Part B, 4 (j), p. 45.

In order to constrain the general market risk of the trading book, we impose an upper bound on the VaR of the trading book. To integrate this VaR constraint into the linear optimization model, as achieved in the section 19.2.2 above, we formulate a second trading book CVaR constraint by the optimization approach by Rockafellar and Uryasev (2000). Therefore, we determine a confidence level α^* such that the α^*-CVaR of the trading book approximates the 99%-VaR of the trading book in the optimal solution $x^\#$, i.e.

$$VaR^{tb}_{99\%}(L^{tb}(x^{tb^\#}, y^{tb})) \approx CVaR^{tb}_{\alpha^*}(L^{tb}(x^{tb^\#}, y^{tb})) \tag{19.18}$$

where $VaR^{tb}_{99\%}(L^{tb}(x^{tb^\#}, y^{tb}))$ denotes the 99%-VaR of the trading book and $CVaR^{tb}_{\alpha^*}(L^{tb}(x^{tb^\#}, y^{tb}))$ the CVaR of the trading book at the confidence level α^*.[33] We model the regulatory capital charge $cap\_charge^{tb}(x^{tb})$ of the trading book as follows:

$$\begin{aligned} cap\_charge^{tb}(x^{tb}) &= spec\_risk(x^{tb}) + mult \cdot CVaR^{tb}_{\alpha^*}(L^{tb}(x^{tb}, y^{tb})) \\ &\approx spec\_risk(x^{tb}) + mult \cdot VaR^{tb}_{99\%}(L^{tb}(x^{tb}, y^{tb})) \end{aligned} \tag{19.19}$$

Let q^{tb} be a real number, $y^{tb}_1, \ldots, y^{tb}_{K^{tb}}$ a sample of market price vectors for the trading book assets, where K^{tb} denotes the number of the market price scenarios of the trading book and $z^{tb}_1, \ldots, z^{tb}_{K^{tb}}$ a set of nonnegative auxiliary variables. We attain the following set of linear inequalities to constrain the general market risk of the trading book, that is approximated in the optimal solution by the left-hand side of the inequality (i) of equation (19.20):

$$\begin{array}{lll} \text{(i)} & q^{tb} + \frac{1}{(1-\alpha^*)} \cdot \frac{1}{K^{tb}} \cdot \sum_{l=1}^{K^{tb}} z^{tb}_1 \leq CVaR^{tb}_{\alpha^*}(L^{tb}(x^{tb}, y^{tb})) & \\ \text{(ii)} & L^{tb}(x^{tb}, y^{tb}_1) - q^{tb} \leq z^{tb}_1, & l = 1, \ldots, K^{tb} \\ \text{(iii)} & -z^{tb}_l \leq 0, & l = 1, \ldots, K^{tb} \\ \text{(iv)} & q^{tb} \in \mathcal{R} & \end{array} \tag{19.20}$$

Regulations require that the market risk of the trading book is limited with respect to the unused components of the tier 1 and the tier 2 capital plus the tier 3 capital available,[34] as illustrated by the second constraint in Figure 19.3 above. We limit the trading book capital charge $cap\_charge^{tb}(x^{tb})$, as defined in equation (19.19), by the eligible regulatory capital components:

$$\begin{aligned} & charge^{tb}(x^{tb}) = spec\_risk(x^{tb}) + mult \cdot CVaR^{tb}_{\alpha^*}(L^{tb}(x^{tb}, y^{tb})) \\ & \leq tier\_3 + (tier\_1\_max - tier\_1\_bb) + (tier\_2\_max - tier\_2\_bb) \end{aligned} \tag{19.21}$$

Additionally, we consider the legal rules that define the relations of the different capital components.[35] We constraint the use of the unused tier 2 plus the tier 3 capital to 250% of the unused tier 1 capital[36] and achieve the following inequality:

$$tier\_3 + (tier\_2\_max - tier\_2\_bb) \leq 2.5 \cdot (tier\_1\_max - tier\_1\_bb) \tag{19.22}$$

[33] We apply an iteration routine that determines a sequence of confidence levels, such that in the approximation the α^*-CVaR and the 99%-VaR of the optimal portfolio $\mathbf{x}^\#$ conincide.
[34] See Basel (1996), Introduction II (a) 1.
[35] See Basel (1988), part I, paragraph 14, p. 4 and Basel (1996), part II (a), paragraph 1, p. 7.
[36] See Basel (1996), Introduction II (a) 1.

We consider upper and lower bounds for the capital components:

$$\begin{array}{ll} \text{(i)} & 0 \leq tier\_1\_bb \leq tier\_1\_max \\ \text{(ii)} & 0 \leq tier\_2\_bb \leq tier\_2\_max \\ \text{(iii)} & 0 \leq tier\_3\_ \leq tier\_3\_max \end{array} \qquad (19.23)$$

According to the legal rules the tier 2 capital is eligible up to 100% of the tier 1 capital.[37] With respect to the optimization model, we need to check with the input data that *tier_2_max* does not exceed *tier_1_max*.[38]

19.2.3.2 Extension of the optimization model with respect to the "Basel II" Capital Accord

In June 1999 the Basel Committee on Banking Supervision (BCBS) released a first proposal to replace the Capital Accord of 1988 with a more risk-sensitive framework, that was revised by a second consultancy paper in 2001 and a third consultancy paper in 2003. The goal of the new framework is to improve the safety and soundness in the financial system. It consists of three pillars, i.e. minimum capital requirements (pillar 1), supervisory review process (pillar 2) and market discipline (pillar 3). With respect to the regulatory capital charge in the optimization model of the RRM Approach we focus on the quantitative requirements of the pillar 1. According to the New Capital Accord the capital adequacy is measured as follows:[39]

$$\frac{\text{tier 1 capital} + \text{tier 2 capital} + \text{tier 3 capital}}{\text{credit risk of the bank book} + 12.5 * (\text{market risk of the trading book} + \text{operational risk})} \geq 8.00\% \qquad (19.24)$$

The numerator of the capital ratio remains unchanged. In the denominator the methods for measuring credit risk are more elaborate than those in the current Accord. The new framework suggests for the first time a measure for operational risk, while the market risk measurement is not affected. Operational risk is defined as "the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events".[40] In the following we describe how the new capital charges for credit and operational risk are integrated into the optimization model.

With respect to measuring *credit risk*, there are two principal options proposed in the New Capital Accord, the standardized and the internal ratings based (IRB) approach. The standardized approach is conceptually the same as the present Accord, but more risk-sensitive. Under the IRB approach, the bank is allowed to use its own internal estimate for the borrowers' credit quality.[41] The results of this risk assessment are translated into estimates of a potential future loss amount and are transformed into more risk-sensitive risk weights than in the standardized approach. We notice that the *structure* of the regulatory capital charge for credit risk as the sum of risk weighted assets is not changed.

[37] See Basel (1988), part I, paragraph 14, p. 4.

[38] We may also check with the input data that the portfolio meets a further regulatory constraint on the tier 2 composition, that limits the long-term subordinated debt in the tier 2 capital to 50% of the tier 1 capital. See Basel (1996), II (a) paragraph 1.

[39] See Basel (2003), part 2, I, paragraph 22, p. 6.

[40] See Basel (2003), part 2, V, A, paragraph 601, p. 120.

[41] The new framework suggests a foundation method and advanced methodologies in the IRB Approach.

We conclude that the basic structure of the capital requirement, that is to assign a linear capital charge weight to the exposure of the bank book assets, is not changed by the New Capital Accord. With respect to the optimization model in the RRM Approach we discern that the basic form of the capital charge for the bank book of equation (19.12) will not be affected; however the input coefficients $reg\_cap = (reg\_cap_1, \ldots, reg\_cap_{n\_bb})'$ *will be more risk-sensitive* according to the new rules of "Basel II."

To calculate *operational risk*, the New Capital Accord suggests alternative methods of increasing sophistication, the Basic Indicator Approach, the Standardised Approach and the Advanced Measurement Approach.[42] The Basic Indicator Approach utilizes one indicator of operational risk for the total of a bank's activity. The Standardised Approach specifies different indicators for different business lines. The operational risk of each business line is measured by an indicator that is multiplied by a "beta" factor, that is defined by the BCBS.[43] The total capital charge is calculated as the summation of the regulatory capital charges of the business lines.

In the Advanced Measurement Approach (AMA) the estimates of operational risk are derived from a bank's internal risk measurement models. The modeling takes into account the special loss risks of the different business lines and estimates the operational loss risk of the bank.[44]

With respect to the RRM Approach we assume that the bank either applies the Standardised Approach or an Advanced Measurement Approach to calculate the regulatory capital charge for operational risk. In both cases, we achieve a *capital charge for operational risk per business line*, either derived by a "beta" weighted indicator in the Standardised Approach or by a risk ratio of an internal model when an Advanced Measurement Approach is applied. We model the operational risk of the bank portfolio as follows. Let op_risk_p denote the capital charge of operational risk of the pth business line, $p = 1, \ldots, P$, where P denotes the total number of profit centers, then the total operational risk op_risk is modeled as

$$op\_risk = \sum_{p=1}^{P} op\_risk_p \qquad (19.25)$$

We point out that the capital charge for the operational risk is not directly linked to the assets of the bank portfolio. We assume that operational risk is managed on the business line level. We conclude that there is no direct link of the regulatory capital charge for operational risk to the decision variables of the optimization model, i.e. the exposure of the assets of the bank portfolio. Therefore, we take into account the operational risk contributions in the RRM Approach as constant input keys.

The New Accord does not address the definitions of the capital elements to cover the different kinds of risk. As tier 3 capital is only eligible to cover market risks,[45] we deduce that operational risk has to be covered by tier 1 and tier 2 capital in the first constraint in connection with the credit risk of the bank book.[46] We modify the above equation (19.13) regarding the capital

[42] Basel (2003), part V, p. 123.
[43] Basel (2003), p. 122.
[44] For the defintion of qualitative and quantitative standards of the AMA, refer to Basel (2003), pp. 121–124.
[45] Basel (1996), Introduction II (a), Definition of capital, p. 7.
[46] This proceeding was confirmed in different discussions with regulators and practitioners; however, we could not find a supporting reference for this interpretation.

charge for bank book risk into

$$cap\_charge^{bb}\,(\mathbf{x}^{bb}) + op\_risk = tier\_1\_bb + tier\_2\_bb \leq tier\_1\_max + tier\_2\_max \qquad (19.26)$$

where we set the input keys $\text{op\_risk}_1, \ldots, \text{op\_risk}_P$ equal to zero when applying the "Basel I" rules. We summarize the modifications in the optimization model due to the New Capital Accord. The constraints defining the capital components eligible to cover risks are not affected. With respect to the capital charge for credit risk, the New Capital Accord will evoke more risk adjusted input coefficients in the vector *reg_cap*, whereas the basic structure of the capital charge for credit risk will remain unchanged. Additionally we take into account operational risk. As operational risk is managed on the business line level, we do not link the operational risk capital charge to the decision variables in the optimization model. We apply a constant to consider the capital charge for operational risk in the first constraint on the bank book risk.

19.2.4 Summary of the optimization model of step 1 of the RRM Approach

In the following we summarize the optimization model (P) of the step 1 of the RRM Approach as derived in the preceding sections, that is consistent with the present "Basel I" and the new "Basel II" Capital Accord.

Optimization model (P)

$$\text{(i)} \qquad \mu(x) = \boldsymbol{\mu}' x = \sum_{j=1}^{n} \mu_j x_j \rightarrow max \qquad (19.27)$$

Constraint 1: Internal risk constraint

$$\begin{array}{ll}
\text{(i)} & q + \frac{1}{1-\alpha} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \leq ec_cap_max \\
\text{(ii)} & L(x, y_{\boldsymbol{k}}) - q \leq z_k, \qquad k = 1, \ldots, K \\
\text{(iii)} & -z_k \leq 0, \qquad k = 1, \ldots, K \\
\text{(iv)} & q \in \mathcal{R}
\end{array}$$

Constraint 2: Regulatory risk constraint

$$\begin{array}{ll}
\text{(v)} & 0 \leq tier_1_bb \leq tier_1_\ max \\
\text{(vi)} & 0 \leq tier_2_bb \leq tier_2_\ max \\
\text{(vii)} & 0 \leq tier_3 \leq tier_3_\ max \\
\text{(viii)} & tier_3 + (tier_2_max - tier_2_bb) \leq 2.5 * (tier_1_max - tier_1_bb) \\
\text{(ix)} & cap_charge^{bb}(x^{bb}) + \ op_risk = tier_1_bb \ + \ tier_2_bb \\
\text{(x)} & spec_risk(x^{tb}) = spec_risk'\ x^{tb} \\
\text{(xi)} & q^{tb} + \frac{1}{(1-\alpha^*)} \cdot \frac{1}{K^{tb}} \sum_{l=1}^{K^{tb}} z_1^{tb} \leq CVaR_{\alpha^*}^{tb}(L^{tb}(x^{tb}, y^{tb})) \\
\text{(xii)} & L^{tb}(x^{tb}, y_l^{tb}) - q^{tb} \leq z_l^{tb}, \quad l = 1, \ldots, K^{tb} \\
\text{(xiii)} & -z_l^{tb} \leq 0, \quad l = 1, \ldots, K^{tb} \\
\text{(xiv)} & q^{tb} \in \mathcal{R} \\
\text{(xv)} & cap_charge^{tb}(x^{tb}) = spec_risk(x^{tb}) + mult \cdot CVaR_{\alpha^*}^{tb}(L^{tb}(x^{tb}, y^{tb})) \\
 & \leq tier_3 + (tier_1_\ max - tier_1_bb) + (tier_2_\ max - tier_2_bb)
\end{array}$$

Constraint 3: Definition of the feasible solutions and of the bank book and trading book vector

(xvi) $$low\_bound \leq \ x \leq up\_bound$$

(xvii) $$\mathbf{x} = \begin{pmatrix} x_1 \\ \cdots \\ x_{n\_bb} \\ x_{n\_bb+1} \\ \cdots \\ x_n \end{pmatrix} = \begin{pmatrix} x_1^{bb} \\ \cdots \\ x_{n\_bb}^{bb} \\ x_1^{tb} \\ \cdots \\ x_{n\_\mathrm{tb}}^{th} \end{pmatrix} = \begin{pmatrix} \mathbf{x}^{bb} \\ \\ \mathbf{x}^{tb} \end{pmatrix}$$

19.3 STEP 2 OF THE RRM APPROACH: RISK RETURN KEYS FOR THE OPTIMUM PORTFOLIO

19.3.1 Survey

An important task of bank management is to set and control targets and budgets for the profit centers, i.e. the business lines, that are managed by objectives. Resources are allocated to the profit centers. These are responsible to attain the planned goals by using the allocated resources and are assessed at the end of the period by their achieved performance. Core objectives from a risk management perspective are target expected returns in combination with risk limits on the economic and the regulatory capital and the risk adjusted performance on these capital resources.

The goal of the RRM Approach is to determine efficient risk return ratios for the bank management. Step 1 of the RRM Approach determines risk return efficient bank portfolios. In step 2 of the RRM Approach we derive profit center risk return objectives, i.e. operational targets for the business lines, from the optimum portfolio of step 1. Step 2 of the RRM Approach provides instruments to implement the overall risk return optimum plan portfolio into operational targets by defining consistent management keys for the profit center.

We first determine these ratios on the asset level and then aggregate them on the business line level: In section 19.3.2 we determine the risk contributions and risk adjusted performance ratios of the assets with respect to the internal and the regulatory risk measurement. In section 19.3.3 we aggregate these ratios on the business line level and thus achieve objectives for the profit center. We define the regulatory risk return ratios in accordance with the present and the new Basel Capital Accord. We summarize the risk return objectives of the RRM Approach in section 19.3.4.

19.3.2 Derivation of risk return keys on the asset level

19.3.2.1 Asset risk contributions and risk adjusted performance with respect to the internal risk measurement

On the bank level, we have constrained the overall risk, measured by CVaR, by the total economic capital available. The internal risk is covered by a corresponding amount of economic capital in order to absorb losses occurring in the bank portfolio. The problem of allocating the overall economic capital *within* the bank portfolio corresponds to the question of determining the contributions of the assets or sub-portfolios to the overall bank portfolio risk. The question of what amount of economic capital is allocated to a sub-portfolio is equivalent to the question of how much internal risk the sub-portfolio contributes to the overall portfolio risk. This problem is non-trivial, as due to diversification effects, the sum of the risks of the sub-portfolios, taken as stand-alone risks, usually is higher than the risk of the overall portfolio. *This advantage of risk reducing portfolio effects needs to be shared fairly between the sub-portfolios*.[47]

Patrik *et al*. (1999) formulate *further properties* that an allocation principle for the internal risk measurement should meet:[48]

- The allocation method should be risk adjusted and account for the riskiness of any sub-portfolio, i.e. consider diversification and dependency from the overall portfolio perspective.
- The capital allocation should be additive, i.e. the overall portfolio risk should be split into the risk contributions of the sub-portfolios which add up to the total portfolio risk.
- The risk contribution of any sub-portfolio is supposed to be independent from its affiliation to an organizational unit.
- The measurement of the risk contribution must be based on the same underlying risk modeling as used for the overall portfolio.
- Furthermore, the allocation should be based on reliable information and also be practicable, i.e. obtainable with a reasonable effort.

Patrik *et al*. (1999) assess alternative methods of capital allocation with respect to these properties, analyzing exposure-based, stand-alone risk-based and global risk contribution measures.[49] *Exposure-based principles* base the capital allocation on the relative exposure of the asset to the overall exposure. They lack the property of risk adjustment. *Stand-alone allocation methods* measure risk contributions by the stand-alone risk of the assets and do not account for portfolio effects. *Global risk measures* define the risk contribution of every sub-portfolio from the overall portfolio perspective and allocate capital to sub-portfolios according to their contribution to the overall portfolio risk. The authors come to the conclusion that allocation methods based on global risk measures best meet the demanded properties of a capital allocation principle.[50]

In an axiomatic approach of the "coherence of risk capital allocation principles" Denault (2001) provides justification for a special global allocation method. By a game-theoretic approach he proves that the *Euler allocation*, that is based on the gradient of the overall risk measure, is theoretically sound and provides fair risk contributions. He models a coalitional game, where players, i.e. sub-portfolios of the bank portfolio, are searching for an optimal split of the overall capital costs of the economic capital. While searching an optimal way to allocate the overall capital cost of the full coalition the players try to minimize their own share of the costs. Denault (2001) shows, that under the assumption of a coherent risk measure, the Euler allocation represents the unique solution of this optimization problem and thus an efficient allocation principle of the overall portfolio risk.

Based on these results we apply the Euler allocation principle to determine the risk contributions of the assets in the bank portfolio. According to the Euler allocation, the risk contribution of the jth asset is defined as the partial derivative of the portfolio risk measure with respect to the jth asset, weighted by the exposure of the jth asset.[51] It can be interpreted as a sensitivity ratio, as the risk contribution of the jth asset answers the question "*how does the overall portfolio risk change with respect to an infinitesimal change of the exposure of the jth asset?*"

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[47] See Denault (2001, p. 2).
[48] See Patrik *et al*. (1999, pp. 63–64).
[49] See Patrik *et al*. (1999, pp. 65–70).
[50] See Patrik *et al*. (1999, pp. 69–70).
[51] See Denault (2001), Patrik *et al.* (1999), Tasche (1999).

Let $\rho(x)$ denote a positively homogeneous risk measure.[52] Assuming that the partial derivatives $\partial\rho(x)/\partial x_j$, $j = 1, \ldots, n$, exist at x, we have, according to Euler's formula the following representation of the overall portfolio risk $\rho(x)$ for the portfolio x:[53]

$$\rho(x) = \nabla\rho(x)'x = \sum_{i=1}^{n} \frac{\partial\rho(x)}{\partial x_i} \cdot x_{\mathrm{i}} \tag{19.28}$$

The risk contribution $\rho_{\mathrm{j}}(x)$ of the jth asset is given by the jth term of $\rho(x)$, i.e.

$$\rho_j(x) = \frac{\partial\rho(x)}{\partial x_j} \cdot x_j,\ j = 1, \ldots, n \tag{19.29}$$

We notice that the Euler allocation splits the overall portfolio risk $\rho(x)$ into *additive* components. At the same time, the risk contributions take into account all *portfolio effects*, as they are defined as sensitivity ratios based on the partial derivatives of the overall portfolio risk with respect to the single assets in the portfolio.

We apply the Euler allocation to the portfolio risk measure CVaR. It can be verified that CVaR satisfies the assumptions to apply the Euler allocation: CVaR meets the required homogeneity property.[54] Tasche (1999) has shown that the partial derivatives of the CVaR exist under quite lenient assumptions.[55] He has identified the jth partial derivative of the portfolio CVaR with respect to the jth asset as the *conditional expected loss of the jth component in the tail of the portfolio loss distribution*. Specifying the general risk measure $\rho(x)$ by the portfolio risk measure of $CVaR_\alpha(L(x, y))$, we achieve according to the Euler allocation the following risk contribution $r_j(x)$ of the jth asset, $j = 1, \ldots, n$, when applying equation (19.29) to $CVaR_\alpha(L(x, y))$:

$$\begin{aligned} r_j(x) &= \frac{\partial\ CVaR_\alpha(L(x, y))}{\partial x_{\mathrm{j}}} \cdot x_j \\ &= E[y_j]x_j - E[y_j x_j \mid L(x, y) \geq VaR_\alpha(L(x, y))], \quad i = 1, \ldots, n \end{aligned} \tag{19.30}$$

The risk contributions $r_j(x)$ of the assets can be easily estimated from a given sample of the market prices as the mean of the losses of the jth asset in the tail of the loss distribution.[56]

We summarize basic properties of the Euler allocation principle. It generates an *additive representation of the overall portfolio risk*, that at the same is risk adjusted and takes into account all diversification effects. It allows us to sum up the risk contributions on any sub-portfolio level and thus to achieve the economic capital allocation for any organizational unit. It is the allocation principle that leads to minimal economic capital costs of any sub-portfolio and also to efficient risk adjusted performance ratios, as is explained in the following. We conclude that the Euler allocation is an appropriate allocation principle for the bank portfolio and therefore is applied in the RRM Approach.

The capital allocation of the economic capital also represents the basis for risk adjusted performance measurement within the bank portfolio. Risk adjusted performance ratios relate the performance to the risk of an asset or portfolio. In general, risk adjusted performance can be defined as a ratio of expected return and the risk contribution.[57] In an analytic approach Tasche

[52] The risk measure $r(x)$ is positively homogeneous (of degree 1), if $\forall t \in \mathcal{R}^+ : r(t \cdot x) = t \cdot r(x)$.
[53] See Litterman (1996) and, for example, Tasche (1999, proposition 3.5, equation (3.6)).
[54] See, for example, Artzner *et al*. (1999) and Tasche (1999).
[55] See Tasche (1999, section 5.3, Lemma 5.6, equation (5.14)).
[56] See Bertsimas *et al*. (2000) and Tasche (1999).
[57] See, for example, Burmester (1999).

(1999) shows that the *Euler allocation is the only allocation principle that is suitable for risk adjusted performance measurement*. The starting point of his observations is the following general allocation strategy:

> *An asset in a portfolio should be weighted higher, if its risk adjusted performance, given by the basic ratio*
>
> $$\textit{risk adjusted return} = \frac{\textit{expected return}}{\textit{risk contribution}} \tag{19.31}$$
>
> *exceeds the overall portfolio risk adjusted performance*.

Tasche (1999) proves, that only if the risk contribution in the denominator of equation (19.31) is measured by the Euler allocation in equation (19.29), we can be sure to improve the overall risk adjusted performance by this allocation strategy. He calls this property of the Euler allocation "suitability for performance measurement". *In other words, if we determine the risk contributions by the Euler allocation principle, we achieve reliable management information, as we measure the "right" risk adjusted performance based on an appropriate risk measure*.

We apply the return on risk adjusted capital, RORAC, as ratio to determine the risk adjusted performance.[58] We define the return on risk adjusted capital $RORAC_j(x)$ of the jth asset as

$$RORAC_j(x) = \frac{\mu_j(x)}{r_j(x)}, \qquad j = 1, \ldots, n \tag{19.32}$$

where the risk contribution $r_j(x)$ is estimated according to the Euler allocation as defined in equation (19.30), and the return contribution $\mu_j(x)$ of the jth asset, $j = 1, \ldots, n$, is given by the jth term of the return function, as defined in equation (19.5) above, i.e.

$$\mu_j(x) = \mu_j x_j, \qquad j = 1, \ldots, n \tag{19.33}$$

19.3.2.2 Asset risk contributions and risk adjusted performance with respect to the regulatory risk measurement

In this section we derive the risk contributions and risk adjusted performance of the single assets with respect to the regulatory risk. We determine the regulatory risk contributions first of the bank book assets and then of the trading book assets.

For the bank book assets we achieve a linear representation. The regulatory risk contribution $reg\_cap_j^{bb}(x^{bb})$ of the jth asset of the bank book corresponds to the jth term of the regulatory capital charge function, as defined in equation (19.12):

$$reg\_cap_j^{bb}(x^{bb}) = reg\_cap_j x_j^{bb}, \qquad j = 1, \ldots, n\_bb \tag{19.34}$$

We notice that these risk contributions are consistent with the present and the new "Basel" rules.[59]

According to the definition of the market risk of the trading book in equation (19.14) we have to consider different components of the regulatory risk contributions of the trading

[58] Further ratios can be applied to measure the risk adjusted performance, differing with respect to the risk adjustment of the expected return in the numerator of the performance ratio. For instance, we might achieve the "risk adjusted return on capital," RAROC, by subtracting the costs of the economic capital in the numerator of the RORAC ratio. For a further discussion of this issue, see, for example, Burmester (1999).

[59] Refer to section 19.2.3.2. We pointed out that the New Capital Accord will require different input data for the vector of risk weights *reg_cap*, while the basic structure of the capital charge function is not affected.

book positions. The risk contribution $spec\_risk_j(x^{tb})$ of the jth asset to the total specific risk is given by the jth term of the corresponding capital charge function, as defined in equation (19.15):

$$spec\_risk_j(x^{tb}) = spec\_risk_j x_j^{tb}, \qquad j = 1, \ldots, n\_tb \tag{19.35}$$

We determine the risk contribution of the jth trading book asset with respect to the general market risk of the trading book as defined by equation (19.17). Applying the Euler allocation on the regulatory value-at-risk of the trading book, the risk contribution $gen\_market\_risk_j(x^{tb})$ of the jth asset to the general market risk of the trading book corresponds to the partial derivative of the VaR of the trading book with respect to the jth asset, multiplied by the exposure of the jth position and the multiplying factor, mult:[60]

$$gen\_market\_risk_j(x^{tb}) = mult \cdot \frac{\partial VaR_\alpha^{tb}(L^{tb}(x^{tb}, y^{tb}))}{\partial x_j^{tb}} x_j^{tb}, \quad j = 1, \ldots, n\_tb \tag{19.36}$$

We sum up both components of the specific and the general market risk contributions to achieve the regulatory risk contributions $reg\_cap_j^{tb}(x^{tb})$ of the jth trading book asset, $j = 1, \ldots, n\_tb$, as

$$reg\_cap_j^{tb}(x^{tb}) = spec\_risk_j(x^{tb}) + gen\_market\_risk_j(x^{tb}),\ j = 1, \ldots, n\_tb \tag{19.37}$$

For a simplified representation of the regulatory risk contributions we use the following notation of the regulatory risk contribution $reg\_cap_j(x)$ of the jth asset of the bank portfolio, $j = 1, \ldots, n$:

$$reg\_cap_j(x) = \begin{cases} reg\_cap_j^{bb}(x^{bb}), & j = 1, \ldots, n\_bb \\ reg\_cap_{j-n\_bb}^{tb}(x^{tb}), & j = n\_bb + 1, \ldots, n \end{cases} \tag{19.38}$$

Considering the new "Basel II" rules, we notice that the definition of the market risk contributions is consistent with the present and the new "Basel" rules, as the capital charge for market risk is not affected by the new rules. Regarding the operational risk we point out that there is no capital charge of operational risk linked to the single assets. Therefore, we do not take into account operational risk contributions on the asset level. We conclude that the definition of the risk contributions as summarized in equation (19.38) is consistent with the present and the new Basel Accord.

In order to calculate the risk adjusted performance of the jth asset, $j = 1, \ldots, n$, with respect to the regulatory capital, we divide its expected return by the regulatory capital used. As this ratio commonly is denoted as the return on (regulatory) equity, we call this ratio $ROE_j(x)$ of the jth asset:[61]

$$ROE_j(x) = \frac{\mu_j(x)}{reg\_cap_j(x)}, \qquad j = 1, \ldots, n \tag{19.39}$$

[60] For the calculation of the risk contributions to the overall value-at-risk, see, for example, Tasche (1999, section 5.2) or Hallerbach (2002, section 4).

[61] The definition of the quotient ROE is not unique in practice. Frequently only the tier 1 capital use is considered in the denominator of the quotient of equation (19.39).

19.3.3 Aggregation of risk return keys on the profit center level

19.3.3.1 Profit center risk contributions and risk adjusted performance with respect to the internal risk

As profit centers are managed by objectives, we aggregate the risk return ratios of the assets on the profit center level to achieve ratios for the enterprise-wide risk return management. We first determine the internal risk contribution of the profit centers and then derive the risk adjusted performance.

As the Euler allocation principle generates additive risk contributions, we simply sum up the risk contributions of the assets affiliated to a profit center to achieve the risk contribution of the corresponding profit center. We achieve the following representation $r_p(x)$ of the risk contribution of the pth profit center, $p = 1, \ldots, P$, where P denotes the total number of profit centers in the bank portfolio, I_p the set of indices of assets affiliated to the pth profit center, $p = 1, \ldots, P$, and $r_j(x)$ the risk contribution of the jth asset, as derived in equation (19.30) above:

$$r_p(x) = \sum_{j \in I_p} r_j(x), \qquad p = 1, \ldots, P \tag{19.40}$$

We aggregate the expected return $\mu_p(x)$ of the pth profit center, $p = 1, \ldots, P$, by adding the expected returns of the assets affiliated to this profit center:

$$\mu_p(x) = \sum_{j \in I_p} \mu_j(x), \qquad p = 1, \ldots, P \tag{19.41}$$

with $\mu_j(x)$ the return contribution of the jth asset, $j \in I_p$, as defined in equation (19.33) above.

We calculate the risk adjusted return $RORAC_p(x)$ of the pth profit center, $p = 1, \ldots, P$, by dividing its expected return by its risk contribution:

$$RORAC_p(x) = \frac{\mu_p(x)}{r_p(x)}, \qquad p = 1, \ldots, P \tag{19.42}$$

19.3.3.2 Profit center risk contributions and risk adjusted performance with respect to the regulatory risk

In order to determine the regulatory risk contribution of the pth profit center, $p = 1, \ldots, P$, we aggregate the risk contributions of the assets affiliated to this profit center. Considering the new "Basle" rules, we additionally take into account the operational risk of the profit centers: let op_risk_p denote the operational risk contribution of the pth profit center, $p = 1, \ldots, P$.[62] We achieve the following representation of the regulatory risk contribution $reg\_cap_p(x)$ of the pth profit center:[63]

$$reg\_cap_p(\mathbf{x}) = \sum_{j \in I_p} reg\_cap_j(x) + op\_risk, \qquad p = 1, \ldots, P \tag{19.43}$$

[62] Refer to section 19.2.3.2 above on how the operational risk is modeled in the RRM Approach.
[63] We set the contributions op_risk$_p$, $p = 1, \ldots, P$, equal to zero with respect to the present "Basel I" rules.

We calculate the return on equity $ROE_p(x)$ of the pth profit center, $p = 1, \ldots, P$, by dividing the expected return by its regulatory risk contribution:

$$ROE_p(x) = \frac{\mu_p(x)}{reg\_cap_p(x)}, \qquad p = 1, \ldots, P \tag{19.44}$$

19.3.4 Summary of the risk return ratios generated by the RRM Approach

In the following we summarize the risk and return ratios as achieved by the RRM Approach. The use of these risk return ratios is illustrated by an application example in section 19.4. We first summarize the ratios on the overall bank portfolio level as achieved from step 1 of the RRM Approach and then of the assets and profit centers, as derived from step 2 of the RRM Approach. Let $x^{\#}$ denote the optimal portfolio of (P) of step 1 of the RRM Approach.

On the bank portfolio level we achieve the following risk return ratios given in Table 19.1 for the portfolio $x^{\#}$. We denote the value of any variable in the optimum solution by an affix "#". We summarize in Table 19.2 the risk return ratios of the assets j, $j = 1, \ldots, n$, of the optimal portfolio $x^{\#}$. We aggregate in Table 19.3 the risk return ratios of the profit center p, $p = 1, \ldots, P$, of the optimal portfolio $x^{\#}$.

Table 19.1 Risk return ratios at the bank level

	Risk return ratios of the optimum portfolio $x^{\#}$	Refer to equation
Expected return	$\mu(x^{\#}) = \boldsymbol{\mu}' x^{\#} = \sum_{i=1}^{n} \mu_i x_i^{\#}$	(19.27)
CVaR	$CVaR_\alpha(L(x^{\#}, y)) = q^{\#} + \frac{1}{(1-\alpha)} \cdot \frac{1}{K} \sum_{j=1}^{K} z_j^{\#}$	(19.27)
Return on risk adjusted capital (RORAC)	$RORAC(x^{\#}) = \dfrac{\mu(x^{\#})}{CVaR_\alpha(L(x^{\#}, y))}$	(19.27)
Regulatory capital	$cap\_charge^{bb}(x^{bb\#}) + cap\_charge^{tb}(x^{tb\#})$	(19.12) (19.19)
Return on equity (ROE)	$ROE(x^{\#}) = \dfrac{\mu(x^{\#})}{cap\_charge^{bb}(x^{bb\#}) + cap\_charge^{tb}(x^{tb\#})}$	(19.27)

Table 19.2 Risk return ratios at the asset level

	Risk return ratios of the assets j, $j = 1, \ldots, n$	Refer to equation
Expected return	$\mu_j(x^{\#}) = \mu_j x_j^{\#}$	(19.33)
Risk contribution	$r_j(x^{\#}) = E[y_j]x_j^{\#} - E[y_j x_j^{\#} \mid L(x^{\#}, y) \geq VaR_\alpha(L(x^{\#}, y))]$	(19.30)
Return on risk adjusted capital (RORAC)	$RORAC_j(x^{\#}) = \dfrac{\mu_j(x^{\#})}{r_j(x^{\#})}$	(19.32)
Regulatory capital	$reg\_cap_j(x^{\#})$	(19.38)
Return on equity (ROE)	$ROE_j(x^{\#}) = \dfrac{\mu_j(x^{\#})}{reg\_cap_j(x^{\#})}$	(19.39)

Table 19.3 Risk return ratios at the profit center level

	Risk return ratios of the profit center p, $p = 1, \ldots, P$	Refer to equation
Expected return	$\mu_p(x^\#) = \sum_{j \in I^p} \mu_j(x^\#)$	(19.41)
Risk contribution	$r_p(x^\#) = \sum_{j \in I^p} r_j(x^\#)$	(19.40)
Return on risk adjusted capital (RORAC)	$RORAC_p(x^\#) = \dfrac{\mu_p(x^\#)}{r_p(x^\#)}$	(19.42)
Regulatory capital	$reg\_cap_p(x^\#) = \sum_{j \in I^p} reg\_cap_j(x^\#) + \ op\_risk_p$	(19.43)
Return on equity (ROE)	$ROE_p(x^\#) = \dfrac{\mu_p(x^\#)}{reg\_cap_p(x^\#)}$	(19.44)

19.4 APPLICATION EXAMPLE

19.4.1 Situation and problem statement

We apply the RRM Approach to an application example in order to illustrate the effects of the risk return optimization and the use of the risk return ratios of the RRM Approach. XY bank wants to generate risk return information for its planning process of the next business year. The bank consists of five profit centers. For simplification we assume that each profit center holds an aggregate position, representing typical bank assets: asset 1 represents high-quality bank bonds (rating AA), asset 2 corporate bonds (rating A), assets 3 and 4 industrial loans of different industries (each of rating B) and asset 5 a trading portfolio that is dependent on an equity index.

The regulatory capital of the bank is used at full capacity and cannot be increased in the next business year.[64] We assume that regulatory capital is charged according to the present "Basel I" rules. The economic capital of 83.17 units is currently used at 99.9%. In this situation the managing board considers to raise the economic capital, i.e. the level of internal risk, by additional reserves of 10.14 units in the next period in order to gain higher profits. The managing board wants to know, whether the application of the additional economic capital will be suitable to increase expected returns in the next period and to improve the portfolio risk return relations such that the bank portfolio will meet the internal hurdle-rate of a portfolio RORAC of 14.00%. Furthermore the management wants to obtain information with respect to the equity strategy of the bank. It should be analyzed how the scarcity of regulatory capital impacts on the risk return optimum plan portfolio and to what extent it prevents additional returns being achieved.

We apply the optimization model (P), as stated in equation (19.27) of the step 1 of the RRM Approach, to the portfolio of XY bank. Additionally, in order to analyze the effects of the scarcity of regulatory capital, we apply the optimization model without the regulatory risk constraint, as formulated by the model (P_{CVaR}) in equation (19.9) to the portfolio of XY bank and compare the resulting risk return ratios. We use the following notation for the different optimization models (P) and (P'):[65]

[64] This is a situation frequently faced in bank management. The regulatory capital basically is determined by the tier 1 capital available, a scarce resource that often is used at full capacity. An increase of the tier 1 capital might require a stock capital increase that needs the approval of the investors and often may not be feasible.

[65] We generate the market price scenarios by an integrated Monte Carlo simulation. We generate vectors of five joint normal distributed random variables based on a given correlation matrix, corresponding to the correlations of the five assets in the bank portfolio. The scenarios of the first four assets are transformed into credit prices of the assets 1 to 4 according to the CreditMetrics

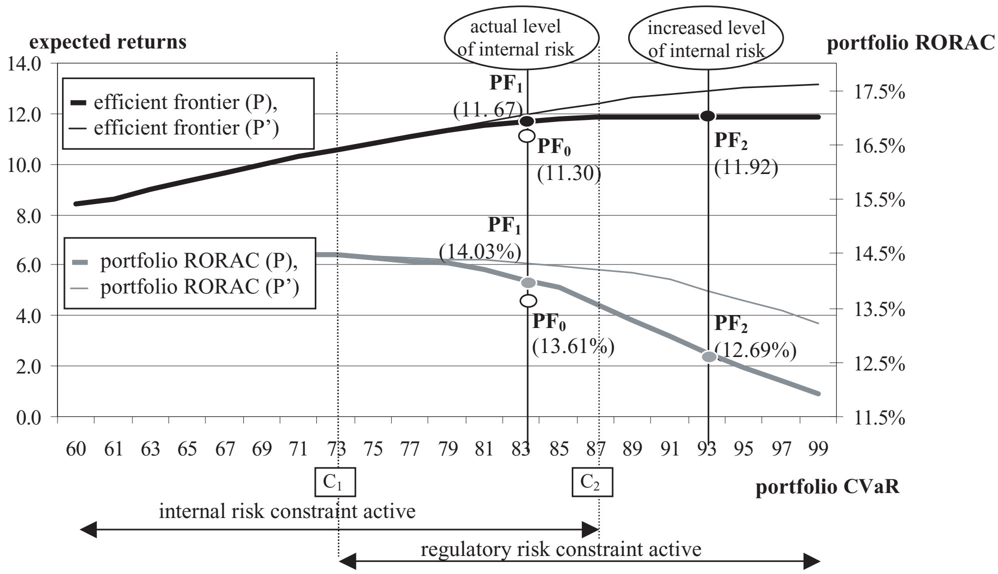

Figure 19.4 Efficient frontiers and portfolio RORACs of the portfolio of XY bank

(P): Maximize expected returns subject to constraints (1), (2) and (3), (i.e. apply (P) including the regulatory risk constraint (see equation (19.27)) to the portfolio of XY bank)

(P'): Maximize expected returns subject to constraints (1) and (3), (i.e. apply (P_{CVaR}) without the regulatory risk constraint (see equation (19.9)) to the portfolio of XY bank).

We pursue the following analyses in order to accomplish the required planning information.

- Apply the optimization model (P) of step 1 of the RRM Approach to generate the efficient frontier of the XY bank portfolio subject to increasing levels of internal risk.
- In order to analyze the effects of the regulatory capital constraint, apply the optimization model (P') to generate the efficient frontier of the portfolio of XY bank *without* the regulatory capital constraint and compare the resulting risk return ratios.
- Apply step 2 of the RRM Approach to analyze the risk return ratios of the profit centers of XY bank. Derive planning information for the business lines and analyze the effects of the regulatory risk constraint.

19.4.2 Results

We apply the optimization models (P) and (P') with increasing levels of the economic capital, i.e. right-hand side constraints of equation (19.27)(ii) and (19.9)(ii) to XY bank portfolio and generate the efficient frontiers of (P) and (P'), as illustrated in Figure 19.4. The initial portfolio is denoted by PF_0, the optimum portfolio at the given level of risk by PF_1 and the optimum portfolio under the increase of the economic capital by PF_2.

Analyzing the efficient frontiers we observe, that to the left of the CVaR level C_1 only the internal risk constraint is active and the regulatory capital is not completely used. To the right

methodology (see JP Morgan, 1997). The scenarios of the 5th asset are transformed into prices of the asset 5 according to the RiskMetrics methodology (see JP Morgan, 1996).

Table 19.4 Risk return ratios of the plan portfolio PF_1

Risk return ratios of the plan portfolio PF_1							Positions		
(1) Profit center	(2) Position	(3) Return target $\mu_p(x^*)$	(4) Econ. cap. limit $r_p(x^*)$	(5) RORAC hurdle rate $RORAC_p(x^*)$	(6) Reg. cap. limit $rec\_cap_p(x^*)$	(7) ROE $ROE_p(x^*)$	(8) Exposure PF_0 x_{j_old}	(9) Exposure PF_1 x_j^*	(10) Change in %
1	x_1	0.05	0.12	43.48%	0.80	6.25%	102.00	50.00	−50.98%
2	x_2	2.46	14.20	17.35%	7.88	31.25%	102.00	98.50	−3.43%
3	x_3	4.80	31.57	15.20%	9.60	50.00%	107.00	120.00	12.15%
4	x_4	4.36	37.29	11.69%	8.72	50.00%	108.00	109.00	0.93%
5	x_5	0.00	0.00	—	0.00	—	6.00	0.00	−100.00%
Bank		11.67	83.17	14.03%	27.00	43.22%			

of the CVaR level C_2 only the regulatory constraint is active, as in this interval we achieve a stationary solution and the economic capital cannot totally be used. In the interval $[C_1, C_2]$ both capital constraints are active, i.e. both capital resources, the economic and the regulatory capital, are maximal applied.

A comparison of the optimum portfolios PF_1 (expected return 11.67) and PF_2 (expected return 11.92) shows that the increase of the economic capital leads to higher absolute expected returns of 0.15 units; however, the RORAC of PF_2 of 12.69% does not meet the internal hurdle rate of 14.00% and is even lower than the RORAC of the initial portfolio PF_0 of 13.61%.[66] Also the economic capital constraint is not active and the economic capital cannot maximal be used in the portfolio PF_2.

We conclude that an increase of the economic capital is not advisable and that the actual level of risk should be maintained. The expected return of the portfolio PF_1 improves the expected return of the initial portfolio PF_0 by 0.37 units. The RORAC of PF_1 of 14.03% complies with the internal hurdle rate. Both capital resources, the economic and the regulatory capital, can be used to their maximum. Analyzing the risk return relations along the efficient frontier we observe that a RORAC maximum portfolio can be achieved at levels of internal risk of the interval [68.9,71.0]. However, the implementation of a RORAC optimizing strategy would require to reduce exposures and expected returns. This might be conflicting with other corporate goals and may not be supported by the shareholders.

Summarizing, we choose PF_1 as plan portfolio for the next business year. From the optimum portfolio PF_1 we derive the risk return profit center objectives. We calculate the expected returns and internal and regulatory risk contributions of the plan portfolio, that serve as the corresponding capital limits for the plan period, and the resulting risk return ratios. We summarize the risk return objectives for the profit centers of XY bank for the next business year in Table 19.4.[67] Let $x^* = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)'$ denote the optimal portfolio PF_1, as given by the exposures in column (9) of Table 19.4.

We analyze the risk return structure of the optimal portfolio PF_1 in more detail. Relating the position changes to their risk contributions and RORACs, we observe that the asset x_1 is reduced by 50.98% despite its highest RORAC, asset 2 with second-best RORAC is slightly decreased, while assets 3 and 4 with lower RORACs are increased. Asset 5 is not included in

[66] We calculate the RORAC ratios of the profit centers according to Table 19.3, row 3.
[67] For a definition of these ratios, refer to Table 19.3 above.

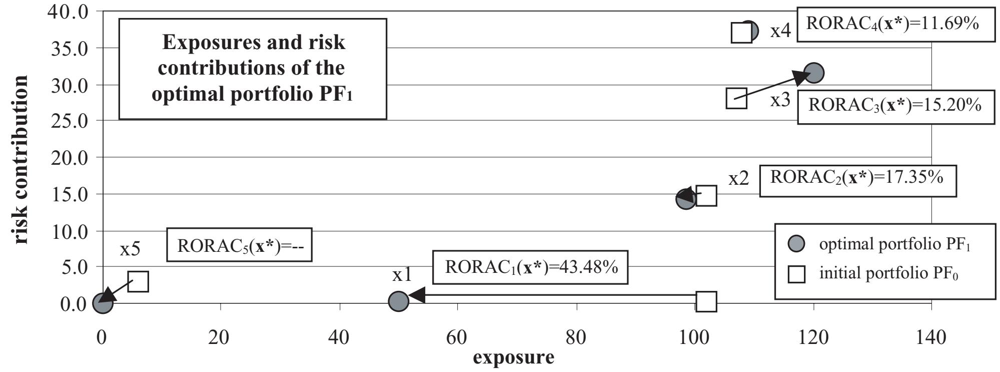

Figure 19.5 Exposures and risk contributions of the optimal portfolio PF_1

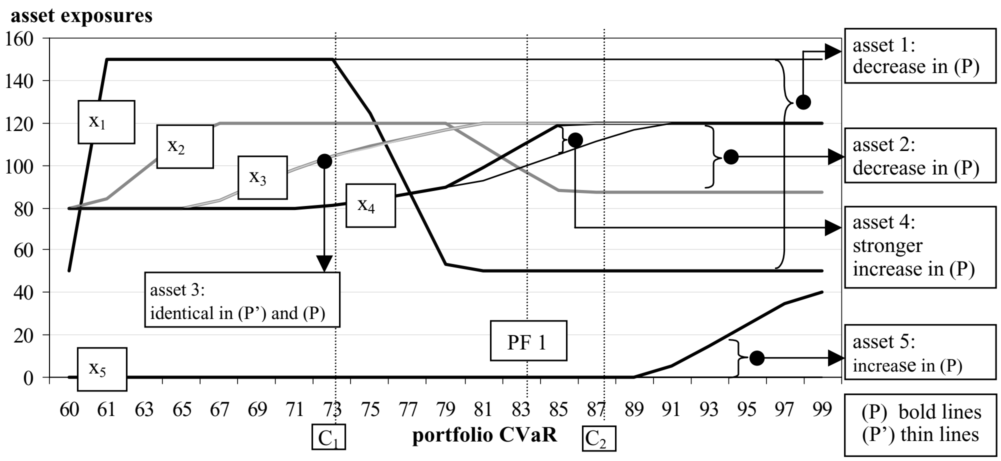

Figure 19.6 Exposures in the optimum solutions of (P) and (P')

the optimum solution. Figure 19.5 relates the exposure to the risk contributions of the initial portfolio PF_0 and the optimal portfolio PF_1.

The fact that assets of higher RORACs are reduced in the optimal solution seems to contradict the intuitive allocation rule to prefer assets of higher RORACs.[68] However, as both capital constraints are active in the optimum portfolio, the weights of the assets are dependent on the risk adjusted return with respect to *both* scarce resources, the economic *and* the regulatory capital. Thus, the effect that assets of higher RORACs are reduced seems to be induced by the scarce regulatory capital.

In order to analyze this effect in more detail, we compare the exposures of the assets in the optimum portfolio with respect to both optimization problems (P) and (P'), as illustrated in Figure 19.6. The bold and thin lines represent the exposures of the assets in the optimum solutions of (P) and (P'), respectively. Starting from the minimum CVaR portfolio, the assets are increased in the optimum solutions in the order of descending RORACs, as defined in

[68] Refer to the allocation rule as described in Section 19.3.2.1, equation (19.31).

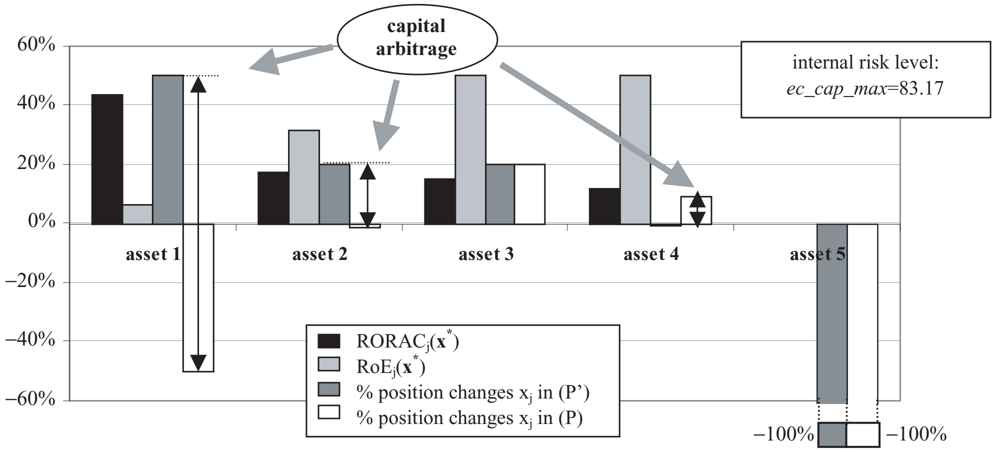

Figure 19.7 Risk return comparison of the optimum portfolios in (P) and (P')

equation (19.32).[69] When the regulatory constraint becomes active at C_1, we observe the effect of *capital arbitrage* or *regulatory arbitrage*, that assets of high RORACs are reduced and assets of higher ROEs are preferred. As a consequence, at any level of risk to the right of C_1, the portfolio risk adjusted performance with respect to the internal risk decreases. With the regulatory risk constraint active, asset 1 is decreased first, followed by asset 2, while asset 4 is increased more strongly. Consequently, if the regulatory constraint is active, at any given CVaR level exceeding the level C_1, we observe lost expected returns in (P) due to the active regulatory risk constraint. This corresponds to our previous observation in Figure 19.4, that the efficient frontier generated by the problem (P) is positioned below the efficient frontier generated by (P') in the interval of an active regulatory capital constraint.

In order to analyze the effect of capital arbitrage more closely, we examine the risk return structure of the optimum portfolio PF_1 at the given CVaR level of 83.17 units in more detail, as illustrated in Figure 19.7. Without the regulatory risk constraint, the exposure of asset 1 of highest RORAC is increased by 47.06%, the one of asset 2 by 17.65% and the one of asset 4 of the lowest RORAC is reduced by 8.08%. In the problem (P), asset 1, yielding the lowest ROE, is reduced by 50.98%. Exposures of assets 3 and 4 with the highest ROE are increased, while the position of asset 2 with higher RORAC but lower ROE than assets 3 and 4 is reduced. As the riskier assets are preferred, the scarce regulatory capital prevents from 0.25 units of additional profits at the given level of internal risk. *It impacts a change in the asset weights towards the riskier assets of lower returns on the economic capital, but higher returns on the regulatory capital.*

These effects of capital arbitrage have been observed and discussed in literature.[70] Our contribution to the analysis of these effects is to provide, by the application of the RRM Approach, a framework to quantify and visualize these effects properly on the basis of suitable internal risk measurement, capital allocation and risk adjusted performance ratios.

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[69] Although the RORACs of the assets of the optimum portfolios x^* differ slightly along the efficient frontier, their ranking is constant in this application example, i.e. $\text{RORAC}_j(x^*) \leq \text{RORAC}_{j-1}(x^*)$, $j = 2, \ldots, 5$.

[70] See, for instance, Koehn and Santomero (1980), Kim and Santomero (1988) and Jackson (1999).

We answer the management's question, how the scarcity of regulatory capital impacts on the bank portfolio and what implications result with respect to the equity strategy of the bank. We summarize that the scarcity of regulatory capital prevents the achievement of 0.25 units of additional expected profits in the risk return optimal plan portfolio PF_1. If the management's primary strategic objective is, for example, a maximization of expected dollar returns or a gain of additional market shares, then an increase of the stock equity may be advisable in order to attain additional absolute returns or market shares in special business fields. If instead maximizing the internal risk adjusted performance is the prior strategic objective, then an increase of the tier 1 capital is not advisable. By an analysis of the efficient frontier of (P) we observed that highest portfolio RORACs can only be achieved at a lower level of internal risk with respect to the current level, where the regulatory risk constraint is not active, i.e. where even the currently available regulatory capital cannot completely be used. For the present portfolio of XY bank, the implementation of a RORAC maximizing strategy would require a reduction of exposures, i.e. market shares, and of absolute dollar returns and might be conflicting with other corporate objectives, e.g. maintaining a target dividend rate per share. Consequently, the management has to decide, how to assess the different corporate objectives. The RRM Approach generates basic information for these decisions.

19.5 CONCLUSION

We have introduced a risk return management approach that generates basic information for a enterprise-wide integrated risk return management. The RRM Approach applies new methods of risk measurement, capital allocation and portfolio optimization to derive a consistent framework of risk return ratios for the bank portfolio. The RRM Approach generates efficient risk return objectives for the bank portfolio and the business lines and can be applied in the planning process of the bank to generate risk return objectives for the overall bank portfolio and the profit centers.

The optimization model of step 1 of the RRM Approach maximizes the expected portfolio return subject to internal and regulatory risk constraints. It is based on the new risk measure of CVaR, which is appropriate for enterprise-wide portfolio risk measurement. The optimization model is consistent with the present and the new Basel Capital Accord. The optimization approach can be applied to generate the efficient frontier of the bank portfolio and allows to determine intervals of maximum use of both capital resources, the available economic and regulatory capital, and of highest portfolio RORACs. The optimization problem can be efficiently solved by linear programming techniques.

In step 2 of the RRM Approach, we translated the optimum portfolio into operational objectives for the profit centers by calculating the risk return keys at the asset level and aggregating them at the business line level. We derived consistent return targets, hurdle rates and capital limits with respect to the regulatory and the internal risk management. We achieved a consistent framework of risk return keys for the bank portfolio.

By an application of the RRM Approach we quantified effects of capital arbitrage. We observed lost profits due to the regulatory risk constraint. We identified sub-optimal portfolios, when assets with higher ROE but higher risk are preferred to assets with lower risk and higher RORACs. Summarizing, the RRM Approach provides basic information for a enterprise-wide risk return management process. In times of increasing risk and decreasing return margins it can contribute to enhance the competitive position of the bank.

Further research will be applied to extend the RRM Approach. Recent research results may be applied for a sophisticated integrated modeling of the market and credit prices to the one-year planning horizon and for the integration of further kinds of risk that are required as input of the optimization model. We will investigate in more detail, how the internal and regulatory risk constraints affect the optimal solutions, when both capital constraints are active. Another point of interest is to develop and apply an optimization algorithm that determines the RORAC-optimum portfolios in one optimization run.

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