

Regulatory Impacts on Risk-Return Efficient Credit Portfolios

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Abstract

Efficient credit portfolio management is a key success factor of bank management. Discussions of the new capital adequacy proposals by the Basle Committee on Banking Supervision enlighten the necessity to consider the credit risk management both from the internal and the regulatory point of view. We introduce an optimization approach for the credit portfolio that maximizes expected returns subject to internal and regulatory risk constraints. With a simplified bank portfolio we examine the impact of the regulatory risk limitation rules on the optimal solutions.

1 Introduction

Efficient credit portfolio management is a key success factor of bank management. In an adverse market environment and intensifying competition banks are exposed to increasing risks and decreasing return margins of their credit portfolio, while bank shareholders are demanding higher risk premiums for their invested capital. The ability to identify risk-return optimal portfolios becomes a fundamental element of credit portfolio management. The recent discussions of the Basle Committee on Banking Supervision enlighten the necessity to manage credit risk simultaneously from an internal and a regulatory perspective.

In this paper, we give a survey of a new optimization algorithm that determines risk-return efficient credit portfolios under internal and regulatory credit risk constraints. We formulate the optimization problem for the credit portfolio based on the risk measure, Conditional Value at Risk, and derive risk-return ratios for the optimal portfolios (chapter 2). With an application example, we analyze the risk-return structure of an optimal portfolio. We examine the impact of the regulatory risk limitation rules and visualize how they may lead to inefficiencies in the credit portfolio management (chapter 3).

2 Optimization Approach

2.1 Definition of the CVaR Risk Measure

The risk measure Value at Risk (VaR), commonly applied in finance, lacks the sub-additivity property, when return distributions are not normal. This means that the diversification of the portfolio may increase portfolio VaR. A similar percentile risk measure, *Conditional Value at Risk* (CVaR) does not have this drawback. The term Conditional Value-at-Risk was introduced in [8]. For continuous distributions, CVaR is equal to the conditional expectation beyond VaR, see [8]. However, for general distributions, it is a weighted average of VaR and the conditional expectation beyond VaR, see [9]. CVaR can be applied to measure loss risk from any asymmetric and discontinuous loss distribution with discrete probabilities and it obeys the property of coherence, see [1,7,9], a set of axioms that a risk measure should meet from the point of view of a regulator [2]. CVaR has been proved to be appropriate for credit portfolio risk measurements [7,8,9].

Let $\mathbf{x}=(x_1, \dots, x_n)'$ be a vector of positions of credit assets of a portfolio, and $\mathbf{y}=(y_1, \dots, y_n)'$ be a vector of the corresponding market prices. For continuous distributions, we define *CVaR deviation* $CVaR_\alpha^\Delta(L(\mathbf{x}, \mathbf{y}))$ of the portfolio loss risk as

$$CVaR_\alpha^\Delta(L(\mathbf{x}, \mathbf{y})) = E[L(\mathbf{x}, \mathbf{y}) | L(\mathbf{x}, \mathbf{y}) \geq VaR_\alpha(L(\mathbf{x}, \mathbf{y}))], \quad (1)$$

where the loss function $L(\mathbf{x}, \mathbf{y})$ is the difference of the uncertain portfolio values and the expected value of the portfolio, i.e. $L(\mathbf{x}, \mathbf{y})=E[\mathbf{y}]'\mathbf{x}-\mathbf{y}'\mathbf{x}$, and $VaR_\alpha(L(\mathbf{x}, \mathbf{y}))$ is the α -quantile of the loss function $L(\mathbf{x}, \mathbf{y})$.¹

2.2 Formulation of the Optimization Model

The optimization problem models basic goals of the credit portfolio management. It maximizes the expected returns of the credit portfolio under internal and regulatory loss risk limits [11]. From the bank's internal perspective credit risks are limited by the economic capital, i. e. the capital resources available that the bank can apply to cover occurring credit losses.² At the same time, the bank needs to limit its credit risk from a regulatory perspective. We consider the loss risk limitation rules set forth by the Basle Committee on Banking Supervision.³ Banks are charged capital to cover the credit risks of their bank book, that is limited by the maximum amount of regulatory capital applicable to cover these risks.⁴ The

¹ In the case of nonzero probability atom at the α -quantile, CVaR is defined as the weighted average of VaR and the conditional expectation beyond the VaR [6].

² The economic capital often is defined as a subset of the bank's equity. Where national law allows the accumulation of hidden reserves, these are commonly applied as elements of the economic capital, as they can be released to cover occurring losses.

³ We are considering the prevailing rules of 'Basle I' [3,4] and will extend our studies by analyzing the effects of the 'new' credit risk weights of the Basle II rules [5].

⁴ We concentrate on the credit portfolio of the bank book. The credit risk of the bank book is limited by the 'tier 1', i.e. the core capital, and the 'tier 2', i.e. the supplementary capi-

capital constraints constrain the absolute expected profits the bank is able to achieve in the planning period. The less economic and regulatory capital is available, i.e. the less risk it is able to take, the less expected profits are achievable in the business period.

We assume a planning horizon of one year and achieve a one period optimization model. The exposures of the assets represent the decision variables. For planning purposes it suffices to consider aggregate positions, e.g. depending on the organizational structure of the bank, product or customer segments, that are accounting to the same profit center.

Let \mathbf{x} be the decision vector and $\boldsymbol{\mu}=(\mu_1,\dots,\mu_n)'$ the vector of the expected returns of single assets. We maximize the expected portfolio return $\mu(\mathbf{x})=\boldsymbol{\mu}'\mathbf{x}$. The internal loss risk is measured by the CVaR deviation of the portfolio loss according to equation (1) and is constrained by the maximal amount of economic capital available, denoted as `ec_cap_max`. Based on the optimization algorithm of Rockafellar/Uryasev [8], the CVaR constraint is approximated by a set of linear constraints, leading to a linear optimization problem. To implement the algorithm, as input data, we use a sample of market price scenarios $\mathbf{y}_1, \dots, \mathbf{y}_K$ of the vector \mathbf{y} .⁵ The regulatory credit risk is measured by the regulatory risk based capital ratios, $\mathbf{reg_cap} = (\text{reg_cap}_1, \dots, \text{reg_cap}_n)'$ and is limited by the available regulatory core and supplementary capital, denoted by `reg_cap_max`. The area of the feasible solutions is defined by upper and lower position bounds, the vectors **low_bound** and **up_bound**. We solve the following linear optimization model:

$$\text{Objective Function } \mu(\mathbf{x}) = \boldsymbol{\mu}'\mathbf{x} = \sum_{j=1}^n \mu_j x_j, \quad (4)$$

Constraint # 1: Internal Risk Constraint

$$\begin{aligned} \text{(i)} \quad & q + \frac{1}{(1-\alpha)} \cdot \frac{1}{K} \sum_{k=1}^K z_k \leq \text{ec_cap_max}, \\ \text{(ii)} \quad & L(\mathbf{x}, \mathbf{y}_k) - q \leq z_k, k = 1, \dots, K, \\ \text{(iii)} \quad & -z_k \leq 0, k = 1, \dots, K, \\ \text{(iv)} \quad & q \in \mathfrak{R} \end{aligned}$$

| |
|---|
| "Internal loss risk (CVaR deviation estimate) ≤ Economic capital" |
|---|

Constraint # 2: Regulatory Risk Constraint

$$\text{(v)} \quad \mathbf{reg_cap}'\mathbf{x} \leq \text{reg_cap_max}.$$

Constraint # 3: Boundaries of the Feasible Solutions

$$\text{(vi)} \quad \mathbf{low_bound} \leq \mathbf{x} \leq \mathbf{up_bound}.$$

tal. The tier 1 capital mainly consists of the core capital of the bank, plus further components. The tier 2 capital includes supplementary capital elements, such as the allowance for loan loss reserves and various long-term debt instruments, such as subordinated debt. See [3], and also [12], p. 119. Our studies will be extended by an integrated analysis of market and credit risk dependent assets under internal and regulatory loss risk limitations.

⁵ In the application example in the next chapter, these market price scenarios are generated by a Monte Carlo-Simulation according to the CreditMetrics approach of J. P. Morgan.

In order to analyze the effects of the regulatory risk constraints on the optimal portfolios, we consider the following optimization models (P') and (P) with and without the regulatory risk constraint, accordingly:

$$\begin{aligned} (P): & \text{ Maximize Objective subject to Constraints \# 1 and 3,} \\ (P'): & \text{ Maximize Objective subject to Constraints \# 1, 2, 3.} \end{aligned} \quad (5)$$

2.3 Risk-Return Analysis of the Portfolio Assets

The contributions of the single assets to the overall portfolio risk and return represent basic information for the risk-return analysis of the optimal portfolios. The return contribution $\mu_j(\mathbf{x})$ of the j -th asset to the portfolio \mathbf{x} is given by the j -th term of the return function, i.e. $\mu_j(\mathbf{x}) = \mu_j x_j$, $j=1, \dots, n$.

We apply the Euler allocation principle to derive the risk contributions of the single assets [3,10]: The risk contribution $r_j(\mathbf{x})$ of the j -th asset is based on the partial derivative of the portfolio risk measure with respect to the j -th asset. It corresponds to the conditional expected loss of the j -th component in the tail of the portfolio loss distribution and can be estimated from the given sample of the market prices as the mean of the losses of the j -th asset in the tail of the loss distribution [10]. We achieve the following risk contribution $r_j(\mathbf{x})$ of the j -th asset, $j=1, \dots, n$:

$$r_j(\mathbf{x}) = \frac{\partial \text{CVaR}_\alpha^\Delta(L(\mathbf{x}, \mathbf{y}))}{\partial x_j} \cdot x_j = E[L_j(\mathbf{x}, \mathbf{y}) \mid L(\mathbf{x}, \mathbf{y}) \geq \text{VaR}_\alpha(L(\mathbf{x}, \mathbf{y}))], \quad (6)$$

$$\text{where } L_j(\mathbf{x}, \mathbf{y}) = E[y_j]x_j - E[y_j x_j \mid L(\mathbf{x}, \mathbf{y}) \geq \text{VaR}_\alpha(L(\mathbf{x}, \mathbf{y}))], j=1, \dots, n.$$

We define the risk-return ratios of single assets, the return on risk adjusted capital RORAC $_j(\mathbf{x})$ of the j -th asset and the return on equity RoE $_j(\mathbf{x})$, i.e. the return on the regulatory capital, of the j -th asset as

$$\begin{aligned} \text{(i) RORAC}_j(\mathbf{x}) &= \frac{\mu_j(\mathbf{x})}{r_j(\mathbf{x})}, & j=1, \dots, n, \\ \text{(ii) RoE}_j(\mathbf{x}) &= \frac{\mu_j}{\text{reg_cap}_j}, & j=1, \dots, n. \end{aligned} \quad (7)$$

3 Application Example

An ABC Bank consists of three typical credit assets: asset 1 represents high quality bonds (Rating AA), asset 2 mortgage loans (Rating BB) and asset 3 retail loans (Rating B). The regulatory capital is used at 94% and cannot be increased in the next business year. The internal risk (CVaR) level may be varied to some extent according to the risk policy of the bank. The initial portfolio uses 48 units of the economic capital. Our goal is to investigate how the risk-return relations of the initial credit portfolio can be improved and how the regulatory risk constraint effects the optimal portfolios. We applied the optimization models (P) and (P') with different CVaR levels. First, we generated the efficient frontiers and analyzed the

overall portfolio risk-return relations. Next, we analyzed the risk-return structures of the single assets of the optimal portfolios.

As shown in the Fig. 1, we observe that the regulatory constraint becomes active at the CVaR-level of 39.9 units. Compared to the initial portfolio, at the given capital levels ($ec_cap_max=48$, $reg_cap_max=10$), the expected portfolio returns can be improved by 0.07 units in (P') and by 0.23 units in (P), keeping the same CVaR deviation level. This means that without the regulatory constraint an additional profit of 0.16 units could be gained. The portfolio RORAC, defined as the expected return $\mu(\mathbf{x})$ divided by the CVaR deviation $CVaR_{\alpha}^{\Delta}(L(\mathbf{x},\mathbf{y}))$ of the portfolio \mathbf{x} , increases from 6.09% to 6.29% in (P') and to 6.63% in (P).

We also observe that the ABC bank can generate higher portfolio RORACs by lowering the level of internal risk and keeping the initial portfolio return. The maximal RORAC of 6.82% can be reached at the interval of $ec_cap_max=[34.9,37.7]$, where the regulatory constraint is not active. However, the implementation of a RORAC optimizing strategy would require reducing the credit volumes and absolute returns. This might be conflicting with other corporate goals and may not be supported by the shareholders.

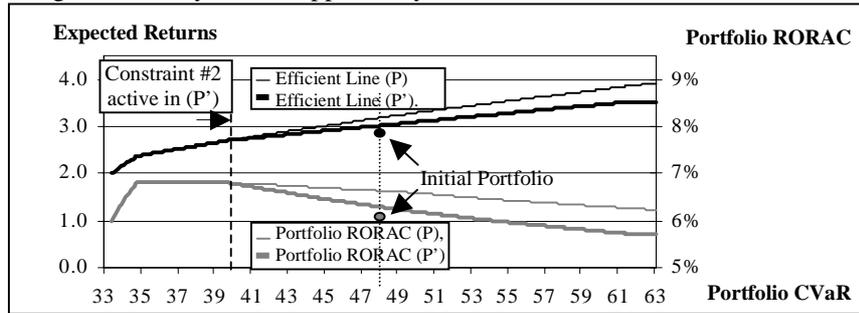


Fig. 1. Efficient Lines and Portfolio RORACs of the Optimization Problems (P) and (P')

In order to analyze the risk-return structure of optimal portfolios we first examine the positions of single assets, which are represented in fig. 2. The narrow and broad lines represent the positions of single assets in the solutions of (P) and (P'), respectively. Starting from the minimal CVaR portfolio, the assets are increased in the optimal solutions in the order of descending RORACs, as defined in the equ. 7 (i).⁶ When the regulatory constraint becomes active, we observe the effect of capital arbitrage: assets with higher RoEs are preferred to assets with higher RORACs, and the overall portfolio level of risk is increased (with the same expected return we can achieve lower risk without regulatory constraint).

⁶ Although the RORACs of the single assets of the optimum portfolios \mathbf{x}^* differ slightly along the efficient line, their ranking remains constant, i.e. $RORAC_3(\mathbf{x}^*) \leq RORAC_2(\mathbf{x}^*) \leq RORAC_1(\mathbf{x}^*)$.

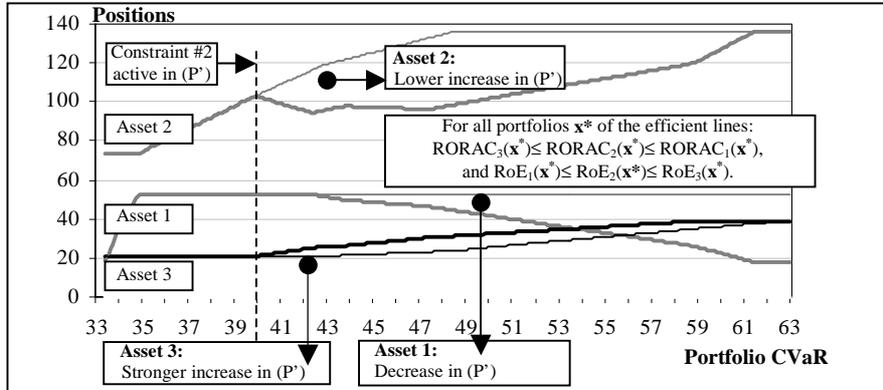


Fig. 2. Impact of the Regulatory Constraint on the Optimal Portfolio Structures

In order to analyze the effect of capital arbitrage more closely, we examine the risk-return structure of the optimal portfolio at the initial CVaR level of 48 units, as described in the fig. 3. Without the regulatory risk constraint, position of asset 1 with highest RORAC is increased by 50%, of asset 2 by 28.3% and position of asset 3 with lowest RORAC is reduced by 21.5%. In (P') asset 1, showing the lowest RoE, is increased less than in (P). Position of asset 3 with the highest RoE is increased, while position of asset 2 with higher RORAC but lower RoE than asset 3 is reduced. The riskier assets are weighted higher in (P'), resulting in lower returns at the given CVaR-level and a sub optimal use of the economic capital, as could be observed in the fig. 1 above.

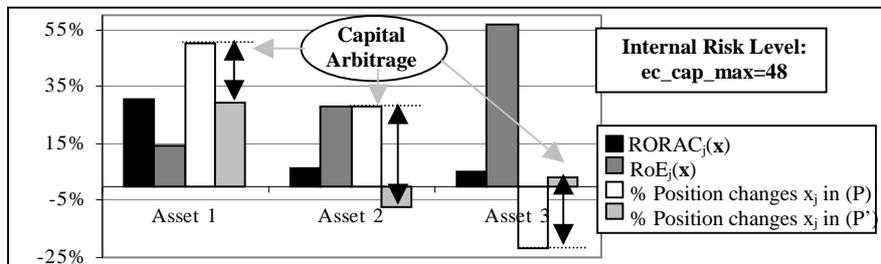


Fig. 3. Portfolio structures of the optimal portfolio in (P) and (P')

4 Conclusion

We have introduced an algorithm that maximizes the expected returns of a credit portfolio subject to the internal and regulatory risk constraints. The algorithm is based on the new risk measure CVaR, which is appropriate for credit portfolio risk measurement, and can be solved by linear programming methods. The optimization model allows calculating intervals of efficient use of both capital resources, the available economic and regulatory capital, and of highest portfolio RORACs. It identifies “unrealized” profits due to the regulatory risk constraint. We con-

ducted risk-return analyses of single assets of the optimal portfolios and found evidence of capital arbitrage, that leads to sub optimal portfolios under the regulatory risk limitation rule, as assets of higher RoE but higher risk are weighted higher than assets of lower risk and higher RORACs.

References

- [1] Acerbi, C. and Tasche, D., On the coherence of expected shortfall. Working paper (2001), can be downloaded from <http://www.gloriamundi.org>.
- [2] Artzner, Ph., Delbaen, F., Eber, J.-M., Heath, D. (1999): Coherent Measures of Risk, *Mathematical Finance*, Vol. 9, No. 3, pp. 203-228.
- [3] Basle committee on Banking Supervision (1988): International convergence of capital measurement and capital standards, Basle, July 1988.
- [4] Basle committee on Banking Supervision committee (1996): Amendment to the capital accord to incorporate market risks, Basle, January 1996.
- [5] Basle Committee on Banking Supervision (2001), Consultative Document: The New Basel Capital Accord, January 2001, Basel, January 2001.
- [6] Patrik, G., Bernegger, S., Rüegg, M.B. (1999): The use of risk adjusted capital to support business decision making, in: Casualty Actuarial Society (Hrsg.), Casualty Actuarial Society Forum, Spring 1999 Edition, Baltimore.
- [7] Pflug, G.Ch. (2000): Some Remarks on the Value-at-Risk and the Conditional-Value-at-Risk, in: Uryasev, S. (Ed.), Probabilistic Constrained Optimization: Methodology and Applications, Kluwer Academic Publishers, pp. 272-281.
- [8] Rockafellar, R. T. and Uryasev, S. (2000): Optimization of Conditional Value-At-Risk, *The Journal of Risk*, Vol. 2, No. 4, pp. 21-51.
- [9] Rockafellar, R.T. and Uryasev, S. (2002): Conditional Value-at-Risk for General Loss Distributions, *Journal of Banking and Finance*, 27/7.
- [10] Tasche, D. (1999): Risk Contributions and Performance Measurement, Working Paper, Technische Universitaet Muenchen.
- [11] Theiler, U. (2002): Optimization Approach for the Risk-Return-Management of the Bank Portfolio, Wiesbaden (In German).
- [12] United States Accounting Office (1998): Risk-Based Capital - Regulatory and Industry Approaches to Capital and Risk, Washington, July 1998.