Dissertation Abstract

Risk Return Optimization of the Bank Portfolio

Ursula Theiler

address:

Risk Training
Carl-Zeiss-Str. 11
D-83052 Bruckmuehl
Germany

Tel.: +49-(0)8062-805545
Fax: +49-(0)8062-805546

http://www.ursula-theiler.de
mailto: theiler@risk-training.org.

Title of the dissertation:
”Integriertes Risk-/Return-Steuerungsverfahren für das Gesamtbank-Portfolio”,
(translated: “Integrated Risk-/Return-Management Approach for the Bank Portfolio”)

Completed: July 2001,
Ludwig-Maximilians-University Munich, Germany,
Faculty of Business Administration and Economics,
Department of Banking

Risk Return Optimization of the Bank Portfolio

Abstract

In an intensifying competition banks are forced to implement enterprise-wide integrated risk return management systems. Financial risks have to be limited and managed from a bank-wide portfolio perspective. Risk management rules must be accomplished from internal and regulatory points of view at the same time. We introduce an optimization algorithm that allows to solve a basic bank-wide risk management decision problem. It maximizes the expected returns of the bank portfolio subject to internal and regulatory loss risk limits. The internal risk constraint is based on the risk measure of Conditional Value at Risk (CVaR), that has been proved to be appropriate for measuring bank-wide loss risk [1,2,3,11]. The regulatory risk constraints represent the prevailing 'Basle Rules' of loss risk limitation [4,5]. We apply an algorithm by Rockafellar/Uryasev to solve the optimization problem [10]. We illustrate the effects of the risk return optimization by an application example of bank management.

Introduction

In an intensifying competition banks are forced to develop and implement enterprise-wide integrated risk return management systems. Financial risks have to be limited and managed from a bank-wide portfolio perspective. Risk management rules must be accomplished from internal and regulatory points of view. We introduce an optimization algorithm that allows to solve a basic bank-wide risk management decision problem. It maximizes the expected returns of the bank portfolio subject to internal and regulatory loss risk limits [13]. The internal risk constraint is based on the risk measure of Conditional Value at Risk (CVaR), that has been proved to be appropriate for measuring bank-wide loss risk [1,2,3,11]. We apply an algorithm by Rockafellar/Uryasev to solve the basic optimization problem [10]. We extend this approach to integrate the bank specific regulatory rules of loss risk limitation. The regulatory risk constraints represent the prevailing 'Basle Rules' of loss risk limitation [4,5,12].

1 Foundations of the Optimization Model

1.A Survey of the Optimization Model for the Bank Portfolio

We pursue an optimization problem that determines risk return efficient bank portfolios to the planning horizon. It maximizes the expected returns of the bank portfolio under internal and regulatory loss risk limits. From an internal perspective, the bank restricts its loss risks to the maximal amount of economic capital, that is internally available to cover occurring losses [15]. At the same time, the bank must comply with external legal loss risk limitations, stated by the 'Basle rules', which constrain the loss risks of the bank portfolio in a regulatory framework and specify limits on the regulatory capital components that are applicable to cover the measured risks [4,5,14].[1] We assume a planning horizon of one year and achieve a one period optimization model. The exposures of the assets represent the decision variables.[2] We achieve the following model structure of the optimization problem (P) for the bank portfolio:

(P)	(1)
objective function: maximize expected returns	
subject to constraints	
constraint 1:	internal risk ≤ economic capital,
constraint 2:	a) regulatory risk ≤ regulatory capital,
	b) constraints on the regulatory capital components,
constraint 3:	definition of the feasible solution: position bounds.

The optimization model maximizes the expected returns of the bank portfolio over the plan period. The economic and the regulatory capital constrain the absolute expected returns achievable in the plan period. The less capital resources are available, the less risk the bank is able to take resulting in lower expected returns in the plan period. Additionally, exposure bounds of the maximal positive and negative exposure changes rare considered in the optimization model by upper and lower position bounds.

1.B Definition of the Internal Risk Measure for the Bank Portfolio

A basic question of the formulation of the optimization model (P) is, how to measure the internal loss risk of the bank portfolio in the constraint 1. An important desirable property of a risk measure is the sub-additivity, that ensures that the risk measure considers portfolio effects reasonably when adding two positions.[3] While the risk measure of Value at Risk, commonly applied in finance for market risk measurement, may lack the elementary property of sub-additivity, if the loss distributions are not normal, the *Conditional Value at Risk* (CVaR), defined as the conditional expectation beyond the Value at Risk, has been proved to be appropriate for risk measurement of any loss distributions [1,2,3,11].

Let $\mathbf{x}=(x_1, \ldots, x_n)'$ be the vector of the positions of the bank assets and $\mathbf{y}=(y_1, \ldots, y_n)'$ the vector of the corresponding market prices. We define the portfolio loss function $L(\mathbf{x,y})$ as the negative deviation of the uncertain from the expected portfolio value at the horizon:[4]

$$L(\mathbf{x,y})=E[\mathbf{y}]'\mathbf{x}-\mathbf{y}'\mathbf{x}. \qquad (2)$$

We define the Conditional Value at Risk $CVaR_\alpha(L(\mathbf{x,y}))$ of the portfolio loss as

$$CVaR_\alpha(L(\mathbf{x,y})) = E[L(\mathbf{x,y}) \mid L(\mathbf{x,y}) \geq VaR_\alpha(L(\mathbf{x,y}))], \qquad (3)$$

where $VaR(L(\mathbf{x,y}))$ is the α-quantile of the loss function $L(\mathbf{x,y})$.[5]

In the above definition CVaR is a sub-additive and convex risk measure that ensures the existence of a risk minimum portfolio on a convex set and thus the solvability of the optimization problem (P). The following figure 1 illustrates a hypothetical portfolio loss distribution and the risk measure of CVaR:

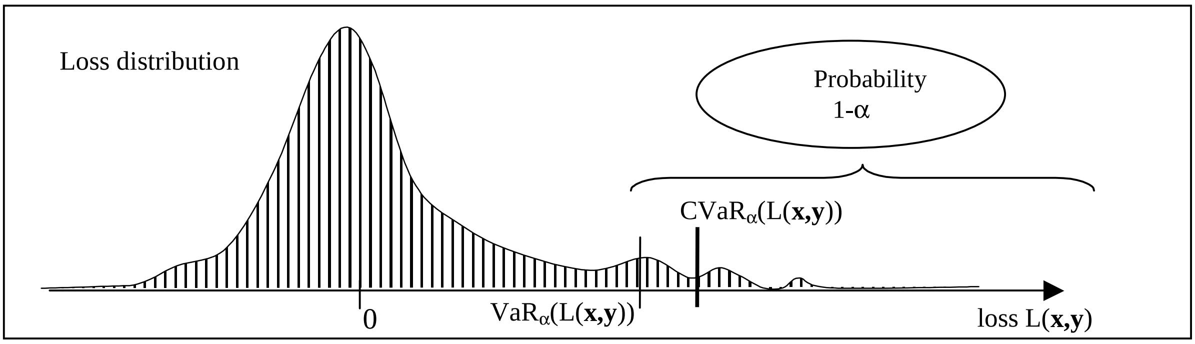

Figure 1: Portfolio Loss Distribution $L(\mathbf{x,y})$ and Corresponding Risk Measures VaR and CVaR

2 Risk Return Optimization Model for the Bank Portfolio

2.A Basic Risk Return Optimization Problem

In a first step, we formulate the basic optimization problem (P') to maximize the expected returns of the portfolio with respect to the constraints 1, that is modeled by a CVaR constraint on the portfolio loss risk, and the constraint 3 defining the feasible solutions:

(P')	(4)
objective function: maximize expected returns	
subject to constraints	
constraint 1:	internal risk ≤ economic capital,
constraint 3:	definition of the feasible solution: position bounds.

Let $\mathbf{x} = (x_1, \ldots, x_n)'$ be the decision variable, i.e. the exposures of the portfolio positions, and $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)'$ the vector of the corresponding expected returns. We define a linear objective function $\mu(\mathbf{x})$ of the expected portfolio return,

$$\mu(\mathbf{x}) = \boldsymbol{\mu}'\mathbf{x} = \sum_{j=1}^{n} \mu_j x_j. \qquad (5)$$

The internal loss risk is measured by CVaR of the loss function $L(\mathbf{x,y})$ and constrained by the maximum amount of economic capital, denoted as ec_cap_max, that is available to cover occurring losses of the bank portfolio.[6] This economic capital framework allows to set business performance into a proper perspective relative to risk [15]. To model the internal CVaR risk constraint, we apply an algorithm by Rockafellar/Uryasev to solve (P') and achieve a set of linear constraints that approximate the original CVaR constraint, as given by the inequalities (6-ii) to (6-v) in the optimization model (P') below [10]. In the optimal solution, the left hand side of the equation (6-ii) approximates the portfolio CVaR. This approach makes use of the fact that CVaR can be expressed in terms of a convex function, that is approximated by a piecewise linear function. Using this approximation, the initial CVaR constraint is replaced by the set of linear constraints (6-ii) to (6-v), that are based on a scenario generation $\mathbf{y_1}, \ldots, \mathbf{y_K}$ of the market prices of the portfolio assets.

In order to define the area of the feasible solutions, we assume upper and lower position bounds of the portfolio assets, denoted as the corresponding vectors **low_bound** and **up_bound** respectively.

By solving the following linear optimization problem (P''), we obtain an approximation of the solution of the (μ,CVaR)-problem (P').[7]

(P'') (6)

(6-i) **Objective Function** $\mu(\mathbf{x}) = \boldsymbol{\mu}'\mathbf{x} = \sum_{j=1}^{n} \mu_j x_j,$

Constraint 1: Internal Risk Constraint

$$(6-\text{ii})\quad q + \frac{1}{(1-\alpha)} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \le ec\_cap\_max,$$

$$(6-\text{iii})\quad L(\mathbf{x},\mathbf{y_k}) - q \le z_k, k = 1,\ldots,K,$$

$$(6-\text{iv})\quad -z_k \le 0, k = 1,\ldots,K,$$

$$(6-\text{v})\quad q \in \Re.$$

Constraint 3: Boundaries of the Feasible Solutions

(6-vi) **low_bound** $\le \mathbf{x} \le$ **up_bound.**

2.B Modeling the Regulatory Risk Constraint

Beneath the internal loss risk limits the bank has to meet regulatory rules of risk limitation, as stated by the Basle Committee on Banking Supervision [4,5].[8] We give a survey, how the prevailing rules are integrated into the optimization model, in order to extend the basic optimization model (P') to the optimization model (P) [13]. According to the Basle Rules, the bank portfolio is separated into the trading book and the bank book. The bank book positions are limited with respect to their credit risk, that is measured by linear coefficients of all bank book assets. We achieve a linear constraint for the credit risk of the bank book, that is limited by the maximum amount of tier 1 and tier 2 capital available.[9] With respect to the trading book positions, regulatory capital is charged for the general market risk and the specific risk of the trading book. We apply a linear constraint for the specific risk. We assume, that the general market risk is measured by a regulatory Value-at-Risk-model [5]. In order to incorporate the resulting VaR-constraint into the CVaR optimization framework, we apply another CVaR constraint on the general market risk of the trading book.[10] The sum of the general market risk and the specific risk of the trading book is limited by the unused tier 1 and tier 2 capital elements of the bank book constraint plus tier 3 capital applicable.[11] Furthermore, regulations constraint the maximum use of the different capital elements and specify the relations between them [4,5]. These constraints are captured in the optimization model by linear constraints.

Summarizing, we achieve a linear one period optimization model to determine risk return optimal bank portfolios, as specified in the problem statement of the equation (1). It is based on the risk measure of CVaR. It basically applies an optimization algorithm by Rockafellar/Uryasev that is extended to incorporate the bank specific regulatory loss risk limitation rules.[12]

3 Application Example

An XY Bank wants to determine its risk return optimal plan portfolio for the next business year under its given capital resources. The bank consists of four typical bank assets: asset 1 represents high quality bank bonds (rating AA), asset 2 corporate bonds (rating A), asset 3 industrial loans (rating B), and asset 4 a trading portfolio that is dependent on an equity index.

In the actual situation, the regulatory capital of the bank is used at 93.8%, and cannot be increased in the next business year. The initial portfolio uses 83.3 units economic capital. In order to gain additional profits, the managing board considers to use additional reserves of 10.1 units to raise the economic capital, i.e. the level of internal risk. The managing board wants to know, if additional economic capital will lead to higher returns in the next business year, and will be suitable to improve the risk return relations in the bank portfolio and to meet the internal risk adjusted hurdle-rate of a portfolio RORAC of 14.0%, where the portfolio RORAC is defined as the expected portfolio return divided by the portfolio CVaR.

In order to provide decision information, we proceed as follows. In order to analyze the effects of the scarce regulatory capital, we define two optimization problems, using the constraint notation of the equation (1) above:

(P_{XY}): Maximize expected returns w.r.t. constraints 1, 2 and 3 (7)

(P^*_{XY}): Maximize expected returns w.r.t. constraints 1 and 3
(i.e. without the regulatory risk constraint).

We generate the efficient lines by running the optimization problem (P_{XY}) and (P^*_{XY}) with different CVaR levels, i.e. values of the maximal economic capital ec_cap_max in the right hand side of the constraint (6-ii), as illustrated in the figure 2 below. The initial portfolio is denoted by PF 0.

The analysis of the efficient lines shows, that to the left of the internal risk level P1 only the internal risk constraint, while to the right of the internal risk level P2 only the regulatory constraint is active. In the interval [P1,P2] both capital constraints are active, i.e. both capital resources are maximal used. Examining the optimal portfolios PF1 and PF2 we observe, that the increase of the economic capital leads to a higher absolute return in PF2, however that its RORAC of 12.69% does not meet the hurdle rate of 14.00% and is even lower than the RORAC of the initial portfolio PF0. Also, in the solution PF2 the economic capital is not maximal used, as the internal risk constraint is not active to the right of P2.

The expected return of the risk return optimal portfolio PF1 improves the expected return of the initial portfolio PF0 by 0.37 units, while maintaining the same level of internal risk. Its portfolio RORAC of 14.03% meets the internal hurdle rate, and it ensures a maximum use of the economic and the regulatory capital.

A maximum RORAC could be achieved in the interval of the internal risk levels of [68.9,71.0]. However, the implementation of a RORAC optimizing strategy would require to reduce market shares, credit exposures and absolute returns. This might be conflicting with other corporate goals and may not be supported by the shareholders. Summarizing, the risk return analyses show that an increase of the economic capital in the next business year does not seem advisable. The actual level of risk should be maintained, and the portfolio PF1 is recommended as plan portfolio for the planning period. In a next step, the optimal plan portfolio needs to be translated into operating return targets and capital budgets for the business lines.[13]

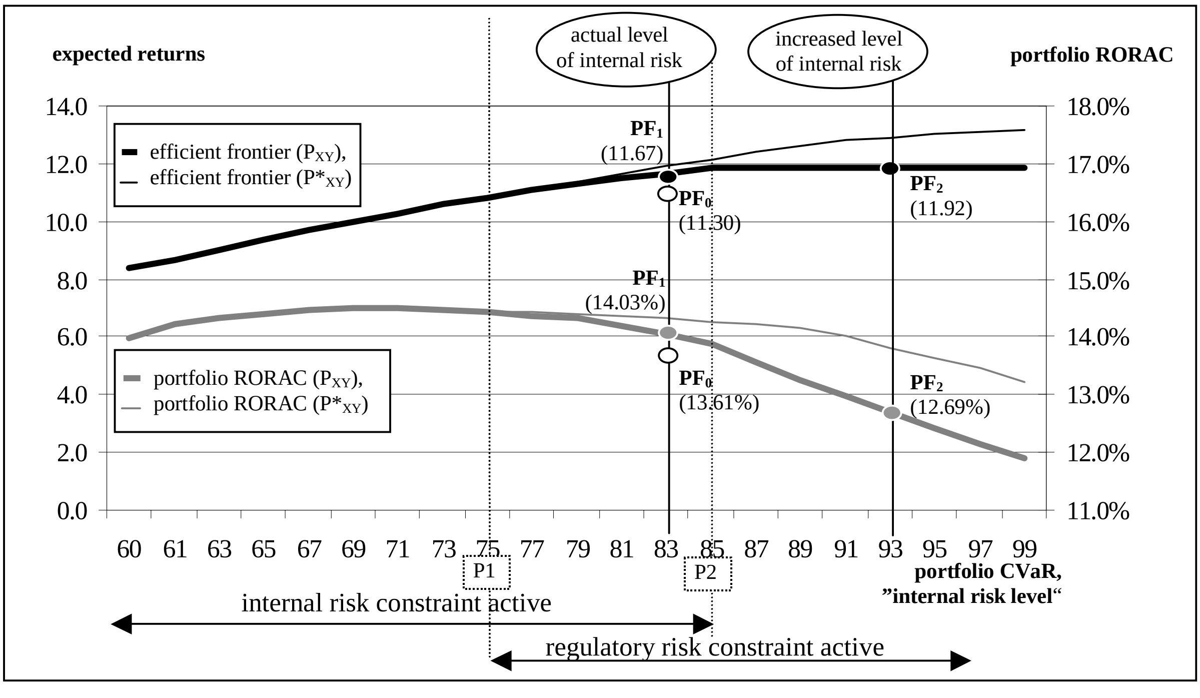

Figure 2: Efficient lines of the XY bank portfolio

4 Conclusion

We have introduced a risk return optimization model that solves a basic decision problem of bank-wide risk management. It maximizes the expected returns of the bank portfolio subject to internal and regulatory loss risk constraints. It is based on the risk measure of CVaR, which is appropriate for bank wide portfolio risk measurement, and can be solved by linear programming methods. By an application of the optimization algorithm we generated the efficient line of the bank portfolio under internal and regulatory loss risk constraints. We determined intervals of efficient use of both capital resources, the available economic and regulatory capital, and of highest portfolio RORACs. Summarizing, the introduced bank portfolio optimization model generates basic information for a bank wide integrated risk return management process.

5 Selected References

[1] Acerbi, C. and Tasche, D. (2002): On the coherence of expected shortfall, Journal *of Banking and Finance*, 26(7), pp. 1519-1533.

[2] Artzner, Philippe; Delbaen, Freddy; Eber, Jean-Marc; Heath, David, Coherent Measures of Risk, in: Mathematical Finance, Vol 9, No. 3, Oxford, UK, July 1999.

[3] Bertsimas, D., Lauprete, G., Samarov, A. (2000): Shortfall as a risk measure, properties, optimizations and applications, Sloan School of Management, MIT, Camebridge.

[4] Basle committee on Banking Supervision (1988): International convergence of capital measurement and capital standards, Basle, July 1988.

[5] Basle committee on Banking Supervision committee (1996): Amendment to the capital accord to incorporate market risks, Basle, January 1996.

[6] Basle Committee on Banking Supervision (2001), Consultative Document: The New Basel Capital Accord, January 2001, Basel, January 2001.

[7] Carey, M., A guide to choosing absolute bank capital requirements, in: Journal of Banking and Finance, 26:5 (2002), pp. 929-951.

[8] Denault, M. (2001), Coherent allocation of risk capital, in: Journal of Risk, Vol. 4, No.1, Fall 2001.

[9] Jorion, P. (2000): Value at risk: The new benchmark for managing financial risk, McGraw-Hill, 2^{nd} ed.

[10] Rockafellar, R. T. and Uryasev, S. (2000): Optimization of Conditional Value-At-Risk, *The Journal of Risk*, Vol. 2, No. 4, pp. 21-51.

[11] Rockafellar, R. T. and Uryasev, S. (2002): Conditional Value-at-Risk for General Loss Distributions, Journal *of Banking and Finance*, 26(7), 2002, pp. 1443-1471.

[12] Tasche, D. (1999): Risk Contributions and Performance Measurement. Working Paper, Technische Universität München.

[13] Theiler, U. (2002): Optimization Approach for the Risk Return Management of the Bank Portfolio, Wiesbaden (Dissertation, in German).

[14] United States Accounting Office (1998): Risk-Based Capital - Regulatory and Industry Approaches to Capital and Risk, Washington, July 1998.

[15] Xiao, J. (2002), Economic capital allocation for credit risk, in: RiskMetrics Group (ed.), RiskMetrics Journal, Volume 2(2), pp. 29-47.

6 Endnotes

[1] We consider the prevailing rules of Basle I [4,5]. The optimization model allows a transition to the new rules of “Basle II” [6].

[2] For planning purposes it suffices to consider aggregate positions, e.g. depending on the organizational structure of the bank, product or customer segments, that are accounting to the same profit center.

[3] Formally, sub-additivity can be defined as follows. The risk measure ρ is sub-additive, if for all sub-portfolios X and Y, $\rho(X+Y) \leq \rho(X) + \rho(Y)$, see [2].

[4] For this definition, see for instance [9], pp. 81 and 109.

[5] In the case of discontinuities at the α-quantile, CVaR can be defined as a weighted average of VaR and the conditional expectation beyond VaR [11].

[6] The economic capital often is defined as a subset of the bank`s equity and may include further equity or loan reserves. Where national law allows the accumulation of hidden reserves, these are commonly applied as elements of the economic capital, as they can be released to cover occurring losses. For a discussion of the definition of the economic capital, see [7].

[7] In this notation, the variable μ represents the expected return of the portfolio.

[8] We consider the prevailing regulatory risk limitation rules. The model also allows a transition to the new Basle II rules [6].

[9] The tier 1 capital mainly consists of the core capital of the bank, plus further components. The tier 2 capital includes supplementary capital elements, such as the allowance for loan loss reserves and various long-term debt instruments, such as subordinated debt. See [4], and also [14], p. 119.

[10] In order to achieve an estimate for the regulatory defined 99%-Value at Risk, we apply a lower confidence level for the CVaR-constraint, such that the CVaR at the lower confidence level approximates the 99%-Value at Risk of the trading book.

[11] The tier 3 capital consists of unsecured subordinated short term debt. See [5], and also [14], p. 122.

[12] An extended formulation of the optimization model is presented in [13], p. 179-181.

[13] For further explanations see [13], pp. 187 - 199. An important issue is to derive internal risk limits of the economic capital for the business lines. This corresponds to the question of estimating the risk contributions of the business lines to the overall portfolio risk of the plan portfolio. The application of the Euler allocation principle [8,12] allows to determine linear risk adjusted capital limits, that add up to the overall portfolio risk and thereby consider all portfolio effects: According to Euler’s formula, the risk contribution of the j-th portfolio position is equal to the j-th partial derivative of the portfolio risk multiplied by the exposure of the j-th asset. The so calculated risk contributions of the single positions can be summed up on any sub portfolio level and be used to determine limits of the economic capital on this level. On the overall portfolio level, the risk contributions add up to the total amount of risk, i.e. of economic capital used by the portfolio considered. See [8,12,13].